A FIRST BOOK

ON

PRACTICAL GEOMETRY

BY

R. V. RAMANAN, B.A., L.T.

GULAM DASTAGIR

Booksellers & Publishers

A FIRST BOOK

OF

PRACTICAL GEOMETRY

BY

R. V. RAMANAN, B.A., L.T.,

(Govt. High School, Bolarum.)

FIRST EDITION 1934

SECOND EDITION 1941

One Rupee.

FOREWORD

I have gone through **A First-book of Practical Geometry** written by my old student and friend Mr. R. V. Ramanan, B.A.,L.T. of the Bolarum High School.

One very striking feature about the book is that almost every problem is preceded by a number of suitable exercises leading up to the constructions used in the problems. Another good feature is that the truths of theoretical geometry as derived from the practical exercises are collected at the end of the book under "Miscellaneous Propositions B." The exercises are copious, well-graded and are drawn from data familiar to the pupils.

These considerations will, I am sure, make the book very useful for the pupils of the Middle Schools in these Dominions.

M. V. Arunachala Sastri, M.A., L.T.

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HYDERABAD
(DECCAN),
22nd June, 1934.

PREFACE

In my experience as teacher of the subject I have found that the text books at present in use do not contain enough exercises to give the drill necessary for the youths in the manipulation of the instruments used in elementary constructions.

It will be admitted by the teachers of mathematics that this practice is essential at the early stages, and that without this training the boys and girls cannot quickly and readily do the constructions, measurements and calculations when they do the study of the theoretical part in the High School classes. An endeavour has been made in this book to supply this want by introducing a large number of exercises.

I have seen that inductive and empirical methods are better suited for teaching Geometry to the young. The exercises have been so arranged as to lead the pupils to arrive at the truths from their own drawings and measurements.

To excite interest local names and places familiar to the learners have been introduced in sums on directions, heights and distances. It is hoped that this book will serve as a text-book preparatory to the theoretical geometry taught in the higher forms, equipped as the pupils will be with a practical knowledge of the truths of most of the theorems.

I shall be thankful for the suggestions that may be offered for improving the usefulness of the book.

BOLARUM HIGH SCHOOL FOR BOYS.
20th June 1934.

R. V. R.

CONTENTS.

CHAPTER I.

I—A POINT.

A dot made by the sharpest pencil is a *point.*

A point is so small that it has no length, breadth or thickness. It has only position but no size or magnitude.

P.

A point is named by one letter, as point P.

Ex. 1. Mark five points on your paper and name them.

Ex. 2. Mark a point A on your paper. Again mark another point B. Join A and B.

II—LINES.

Mark two points on your paper. Call them P and Q. Join them with a sharp pencil. This is called a *line.*

A line joins two points.

A line is named by two letters, as, the line AB; the line C D. A line has only length but no breadth.

Fig. 1. 

Fig. 2. 

You can join two points in two different way either directly as in figure 1, or crookedly as figure 2.

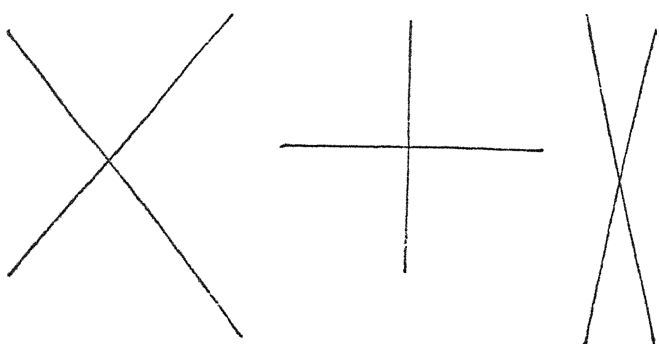
In fig. 1. AB is a straight line. The line joining the two points does not change in direction.

A straight line joins two points without any change in direction.

In Fig. 2. CD is a curved line. The line joining the two points changes in direction.

A *curved line* joining two points changes in direction.

You can go to your school from your house by several roads. But you will find that the straight road is the shortest one. So also a straight line is the shortest distance between two points.



In each of these figures you find that two straight lines cut one another at only one point.

Properties of Straight Lines.

1. A straight line does not change in direction.

2. It is the shortest distance between two points.

3. Two straight lines can cross one another at only one point. A point is correctly marked by crossing two small straight lines. P and Q are points.

P × Q ×

III—STRAIGHT LINES AND THEIR MEASUREMENTS.

Straight lines are drawn with the ruler or the scale found in your mathematical instrument box.

On one edge of the scale you have inches marked and on the other centimetres.

Each inch is divided into ten equal parts. Each division of an inch is equal to $\frac{1}{10}$ or ·1 of an inch.

Each centimetre too is divided into ten equal parts. Each division of a centimetre is equal to $\frac{1}{10}$ or ·1 of a centimetre or one millimetre.

If a line is 5 inches and five divisions long it is read five decimal five inches, and written 5·5″.

If the line is six centimetres and seven divisions long, it is read six decimal seven centimetres, and written 6·7 cm.

Ex. 1. Measure the following lines first in inches, then in centimetres and write down the length of each.

TABLES.

British System.

12 Inches (in. or″)	*make*	*1 Foot (ft. or′)*
3 Feet	,,	*1 Yard (yd.)*
220 Yards	,,	*1 Furlong (fur.)*
8 Furlongs	,,	*1 Mile*

French or Metric System.

10 Millimetres (mm.)	*make*	*1 Centimetre (cm.)*
10 Centimetres	,,	*1 Decimetre.*
10 Decimetres	,,	*1 Metre.*
10 Metres	,,	*1 Dekametre.*
10 Dekametres	,,	*1 Hectometre.*
10 Hectometres	,,	*1 Kilometre.*

1 cm.=·39 in. (nearly)

Ex. 2. With the help of a foot rule (a measure which is a foot long), divided into inches, find

(i) the length of the long side of a desk,
(ii) the length of the class room,
(iii) the height of a door,
(iv) the breadth of a window.

Ex. 3. With the help of a yard measure, divided into feet and inches, find

(i) the length and breadth of a piece of cloth,

(ii) the length of the school hall,

(iii) the breadth of the school verandah.

Ex. 4. With the help of a measuring tape, divided into feet and inches, find

(i) the length of the foot-ball ground,

(ii) the length and breadth of the badminton court,

(iii) the length of the school compound wall,

(iv) the distances of any two houses from the school gate.

Ex. 5. Mark a point P on the paper. From P draw a straight line PQ 6 cm. in length.

Ex. 6. Draw a straight line AB 2·8" long. Then make AB one inch longer.

Ex. 7. Mark a point O. From O draw four straight lines OA, OB, OC and OD in different directions, of lengths 2·3", 2", 2·9 cm. 5·4 cm. respectively.

Ex. 8. Draw a straight line AB 3·6 cm. long. Produce AB to any point C. Measure BC. Add the lengths of AB and BC. Measure AC. What do you notice?

Ex. 9. Draw a straight line PQ of length 3·8 cm. Produce PQ to R so that the length QR is 2·2 cm. Measure PR.

Ex. 10. Draw a straight line AB 5" long. Mark a point X in AB such that AX=3·8" in length. Measure XB.

Ex. 11. Draw a straight line MN of length 7·6 cm. Mark a point X in MN such that MX=2 cm. in length. Again mark another point Y in MN such that NY=3·3 cm. in length. Find the length of XY and verify your result by measurement.

Ex. 12. Draw a straight line CD=3·6" in length. In CD mark a point X at a distance of 2·8" from D. Mark also another point Y in CD at a distance of 2" from C. How far is Y from X? Verify your calculation by measurement.

Ex. 13. Mark a point X. Mark another point Y at a distance of 4 cm. from X.

Ex. 14. Mark a point A and another point B at a distance of 5·5 cm. from A. Join AB. In AB find two points P and Q such that the length of AP=2 cm. and that of PQ=2·8 cm. How far is B from Q?

Ex. 15. H is 12 miles from B. S is on the way. It is $7\frac{1}{2}$ miles from B. If the three places are on a straight road draw a figure on your paper showing the three places. How far is H from S? Verify by measurement. (Take 1 mile=1 cm. in your figure).

Ex. 16. One metre=one yard and 3·4". Express 6 cm. in inches.

Ex. 17. In your scale an inch is divided into tenths. In my scale an inch is divided into eigths. You measure a straight line and find it to be 2·5". What number will express the same length if measured with my scale?

Ex. 18. The map of Europe is drawn to the scale of 25 miles to an inch. If on that map you find the distance from London to Oxford to be 2·5" what is the actual distance between the two cities?

Ex. 19. The Nizam's State Railway map is drawn to the scale one inch to 32 miles. If on that map Secunderabad is 1·6" from Vikarabad, what is the actual distance between the two stations?

Ex. 20. Karimnagar is 40 miles from Kazipet. What distance will be shown on the map between the two places if the scale is 16 miles to one inch? Draw a straight line showing the distance.

Ex. 21. The distance from Rangoon to Colombo is 1,230 miles. What length on a map of the Indian Empire drawn to the scale 1 cm. to 100 miles will express the distance between the two places? Draw a straight line showing the distance.

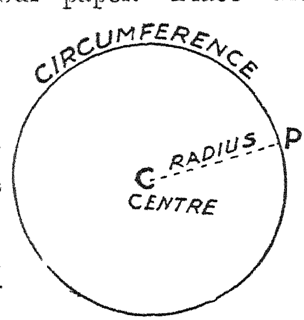
Ex. 22. Secunderabad is 120 miles from Wadi. What length will express the distance between the two places on a map drawn to scale 1 cm. = 8 miles? Draw a straight line to represent the distance between the two places.

CHAPTER II.

CIRCLES.

Fit a sharp pencil to one arm of the compasses. See that the point of the pencil and the steel point are in the same level.

Mark a point C on your paper. Place the steel point at C. Keeping this steady, turn the pencil-arm until the pencil point returns to its first position. A curved line is traced by the point of the pencil. The figure drawn is a circle.



The point C is called the *centre* of the circle.

The curved line is called the *circumference.*

The distance between the needle point and the pencil point is called the radius of the circle.

Mark a point P on the circumference. Join CP. Measure CP. This is the length of the radius of the circle.

So the *radius* of a circle is the distance of any point on the circumference from the centre.

Radius (*Singular*) Radii (*Plural*)

Ex. 1. (*a*) Mark a point O on your paper. Take a radius of 2" in your compasses. With centre O draw a circle.

(*b*) Mark points A, B, C and D on the circumference. Join these points to O. Measure OA, OB, OC and OD. What do you notice ?

Ex. 2. Mark a point O on your paper. With centre O and radius = 2·4 cm. describe a circle. On the circumference mark several points. Measure the distance of each point from the centre. What do you notice ? What do you call these lengths ?

You find that the radii of a circle are equal.

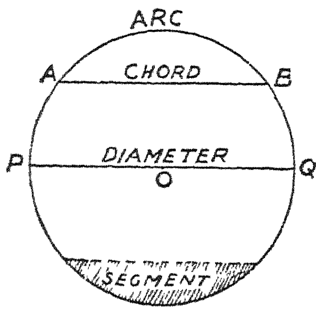
Ex. 3. Describe a circle of radius 4·2 cm. Name the centre P. Mark four points, each being at a distance of 4·2 cm. from P. Where do the points lie ?

Ex. 4. Mark a point C on your paper. Mark another point D at a distance of 2·3" from C. Mark four more points equally distant from C. On what line do these points lie ?

Ex. 5. Draw a straight line AB = 2·8 cm. in length. With A as centre and radius 2 cm. draw a circle. Again with centre B and radius 2·3 cm. draw a cirle. What do you notice ?

Ex. 6 Mark a point O on your paper. With centre O draw a circle of radius 3 cm. Again with the same centre draw circles of radii 4 cm., 4·4 cm. and 5 cm.

Do these circles cut one another? These circles have the same centre, but have radii of different lengths. They are called *concentric circles.*



Mark a point O. With O as centre and radius equal to 4 cm., draw a circle.

Mark two points A and B on the circumference.

The part AB of the circumference is called an *arc.*

An *arc* is any part of the circumference of a circle.

Join AB. The straight line AB is called a *chord.*

A *chord* joins the ends of an arc. The figure bounded by an arc and a chord is a *segment* of a circle. (segment = a part).

Take a point P on the circumference. Join PO and produce it to meet the circumference again at Q. PQ is called a *diameter* of the circle.

A *diameter* of a circle passes through the centre and is cut off by the circumference at both ends.

A diameter of a circle is twice the radius.

Ex. 7. Draw a circle of any radius. Draw a diameter in it. Cut out the circular piece and fold it along the diameter. What do you notice?

Ex. 8. Repeat Ex. 7, taking circles of different radii. What do you learn from your results?

A diameter divides a circle into two equal parts. Each part is called a *semi-circle*. (*Semi* means *half*).

A semi-circle is a part of a circle bounded by the diameter and the arc cut off by it.

Ex. 9. Describe a circle of radius 2·6 cm. In that circle draw a chord and a diameter.

Ex. 10. In a circle of radius 5 cm. mark the following :—

(i) Centre (ii) a radius (iii) circumference (iv) an arc (v) a chord (vi) a diameter.

Ex. 11. Describe a circle of radius 2·1". On the circumference mark four arcs and name them.

Ex. 12. Mark a point O. With centre O draw an arc of a circle of radius 3 cm.

Ex. 13. Describe a circle of diameter 4".

Ex. 14. Describe semi-circles of radii (i) 1·2" (ii) 2 cm. (iii) 2·5" (iv) 1·6" (v) 3·7 cm.

Ex. 15. Describe semi-circles on diameters—(i) 2·3 " (ii) 3·6 cm. (iii) 2·9 " (iv) 5·5 cm.

Ex. 16. Draw a straight line AB of length 6·2 cm. With centre A and radius equal to 4 cm., draw an arc above AB. Again with centre B and radius 3·5 cm., draw an arc on the same side to cut the first arc at P. Join AP and BP and measure their lengths.

Ex. 17. Draw a straight line PQ 7 cm. long. With centre P and radius equal to 4 cm. draw an arc. Again with centre Q and the same radius draw another arc to cut the former arc at O. How far is O from A and B ?

Ex. 18. Draw a straight line XY equal to 2·8" in length. With centre X and radius 1·5" draw a circle. With centre Y and radius 1·8" draw another circle. Name the points, where they cut, P and Q.

Ex. 19. Draw a straight line AB of length 2". With centre A and radius equal to 1·5 in. draw a circle. With B as centre and with the same radius draw another circle. At how many points do the circumferences cut ?

Ex. 20. Draw a straight line MN 3" long. With centre M and radius 1·5" draw a circle. Again with N as centre and with the same radius draw another circle. Do the circumferences cut or touch ?

Ex. 21. Draw a straight line RS 7 cm. long. With centre R and radius equal to 3 cm. draw a circle. Again with centre S and with the same radius draw another circle. Do the circumferences cut at two points ?

Ex. 22. A and B are two points 6 cm. apart. With centres A and B describe circles of radii—

(i) 4 cm. and 3 cm., (ii) 3 cm. and 3 cm., (iii) 3 cm. and 2 cm. At how many points do the circumferences cut in each case? What do you notice in the third case? Draw a separate figure in each case.

Ex. 23. Mark two points X and Y 2" apart. With centres X and Y draw the following pairs of circles, the radii of which are—(i) 1·6" and 1" (ii) 1" and 1" (iii) 1" and ·8". Which of these circles meet? What is the sum of each pair of radii?

What do you learn from Ex. 22 and 23?

Ex. 24. Without actually drawing find which of the following pairs of circles meet, and at how many points. The centres are always 5 cm. apart. The radii are (i) 3 cm. and 4 cm. (ii) 4 cm. and 4 cm. (iii) 2·5 cm. and 2·5 cm. (iv) 2 cm. and 3 cm. (v) 3·5 cm. and 1·5 cm. (vi) 2 cm. and 1·5 cm. (vii) 2 cm. and 2 cm. (viii) 1·5 cm. and 2·5 cm.

Ex. 25. Draw a straight line AB 2·5 in. long. With centres A and B draw circles of equal radii so that the circumferences may cut one another.

Ex. 26. Two points P and Q are 5 cm. apart. Find another point O which is 4 cm. from P and 6 cm. from Q.

Ex. 27. Two points A and B are 2·7 in. apart. Find a point P outside AB so that PA is equal to PB.

Ex. 28. On a straight road two houses X and Y are 500 yards apart. A well is 350 yards from X and 400 yards from Y. Draw a plan to show the positions of the three places. (Scale 50 yards = 1 cm.)

Ex. 29. Your school is 400 yards from your house. Find the position of the railway station which is 300 yards from the school and 200 yards from the house. (Scale 100 yards = 1").

Ex. 30. Two points M and N are 7 cm. apart. Find a point A distant 7 cm. from each of the points M and N. Join AM, MN and AN. Describe semi-circles on AM, MN and AN.

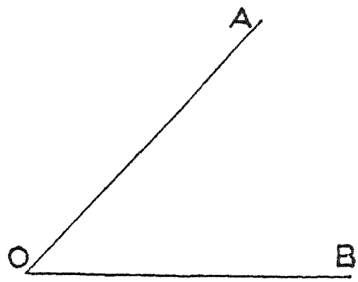
Ex. 31. Draw a circle of radius 5 cm. Mark a point P on the circumference. With P as centre and radius 3 cm. draw an arc to cut the circumference at Q. Join PQ. Find the length of the chord PQ.

Ex. 32. Draw a circle of radius 1·7 in. In it draw chords of lengths 1", 2", 1·5", ·7".

CHAPTER III.

ANGLES.

Draw a straight line OA. From O draw another straight line OB.



AOB is a called an *angle*.

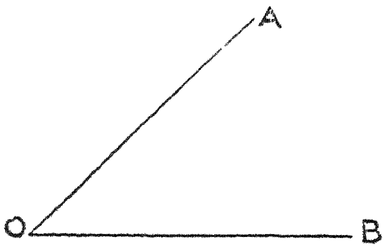
Ex. 1. Draw a straight line XY 2" long. From X draw another straight line XZ of length 1". What is ZXY called?

If two straight lines meet at a point *an angle* is formed.

Ex. 2. Mark a point O on your paper. From O draw four lines OX, OY, OZ and OM in different directions. How many angles are formed? Name them.

Ex. 3. Draw two straight lines XY and PQ cutting one another at O. How many angles are there? Name them.

Ex. 4. Draw a circle of any radius. Call the centre O. Mark on the circumference points A, B, C, D, E and F. Join them to the centre. Name the angles formed.



AOB is an angle.

The lines OA and OB are called the *arms* of the angle.

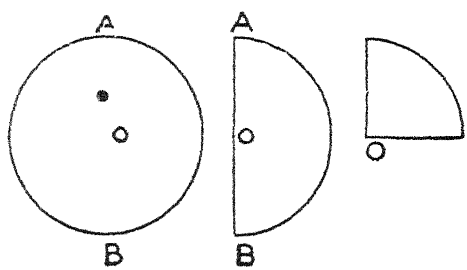
O is called the *vertex*.

Plural of vertex is *vertices*.

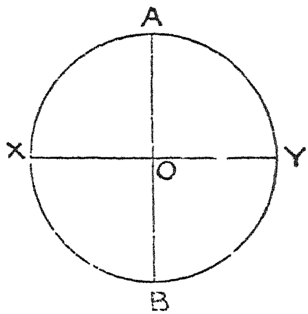
An angle is named by three letters, as 'the angle AOB' or 'the angle BOA'; if there is only one angle at a point, it may also be expressed as 'the angle O.'

Ex. 5. Mark a point X. From X draw two lines XY and XZ. Name the angle, the arms and the vertex.

Draw a circle on a piece of paper. Cut out the circular piece fig (i) and fold it along a diameter into two halves fig (ii). Again fold the semi-circular piece into two halves as shown in the figure (iii).



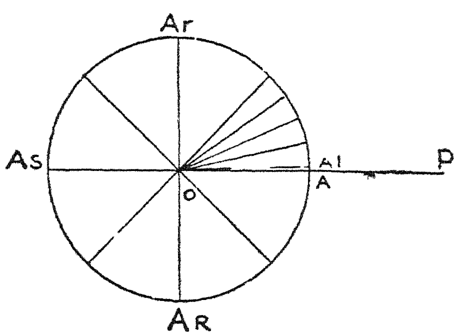
Then unfold the paper and draw lines along the creases.



You see two diameters crossing one another. Four equal angles are formed. Each angle is called a right angle. AOX is a right angle. Name the other right angles.

The minute hand and the hour hand of a clock at any time make an angle.

At 12 o'clock the minute-hand is over the hour-hand. Then the size (or magnitude) of the angle between the hands is zero. At 3 o'clock the angle made by the hands is a right angle.



OA and OP are two straight lines. OA is over OP.

If OA revolves about O it makes in different positions the angles POA_1, POA_2, etc., and the point A traces out a circle with O as centre.

When OA overlaps OP, the magnitude of the angle between OA and OP is zero. POA_r is a right angle. POA_s contains two right angles, and POA_R three right angles.

If the arc AA_r is divided into 90 equal parts A_1 being the first division, the size of the angle POA_1 is said to be 1 degree (1°).

A right angle contains 90°.

Ex. 6. How many degrees are there in—(i) POA_s (ii) POA_r (iii) POA_R.

Ex. 7. How many degrees are there between the hour-hand and the minute-hand of a clock at the following times—(i) 3 (ii) 6 (iii) 9 (iv) 4 (v) 7 (vi) 8 (vii) 10 (viii) 11.

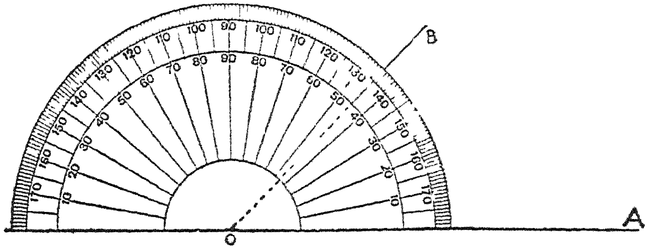
The sign for an angle is $\llcorner$ or $\wedge$.

Angle AOB may be written as $\llcorner AOB$ or $A\hat{O}B$.

An angle is measured with a protractor. This is the semi-circular piece in your mathematical instrument box. The semi-circular arc is divided into 180 equal parts. Each small mark in the arc is a degree (1°).

The middle point of the diameter of the piece is also marked. This is the centre of the protractor.

(*a*) Measure the angle AOB.



Place the straight edge of the protractor on OA so that the centre of it is at O. Look at the protractor and find which mark of it coincides with the line OB or OB produced. The degree marks must be counted from the side on which OA lies.

(*b*) Make an angle AOB = 45°.

Draw a straight line OA. OA is one arm of the angle. Place the straight edge of the protractor in a line with OA. The centre must be at O. Then mark a point on your paper close to the 45th mark on the protractor. The 45th division mark must be counted from the side on which the first arm OA lies. Call this marked point B. Join OB. AOB is an angle of 45 degrees (45°).

The number of degrees in any angle does not change if the arms are lengthened or shortened. So, whenever the arm of an angle is too short to reach the rim of the protractor, used to measure the angle produce the arm.

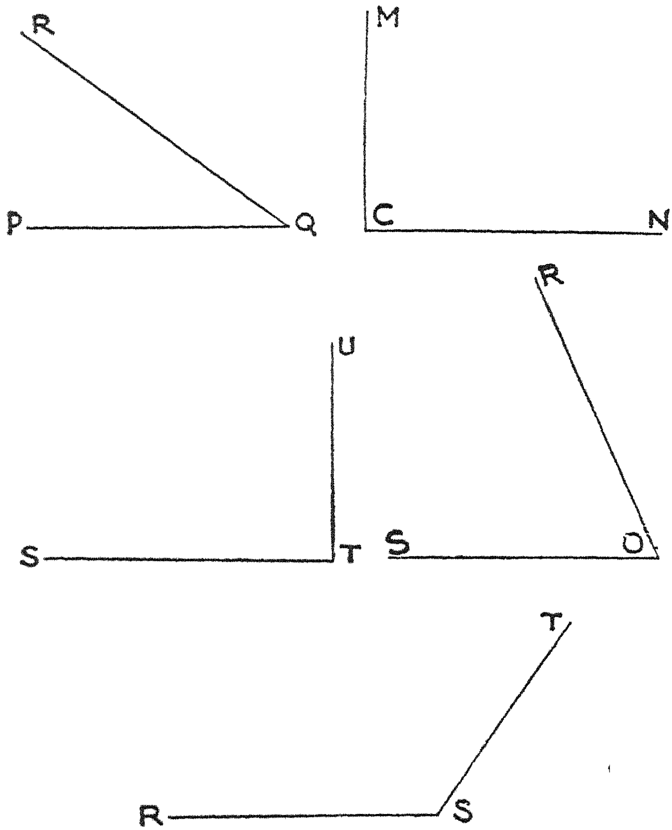
A *right angle* contains 90°.

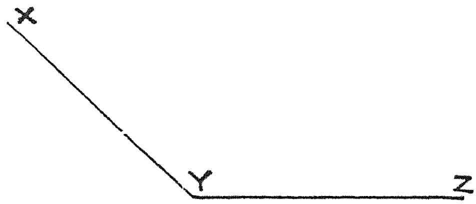
An angle smaller than a right angle is called an *acute angle*.

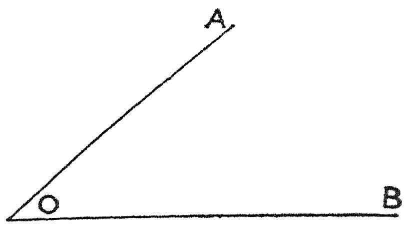
An angle greater than a right angle but less than two is an *obtuse angle.*

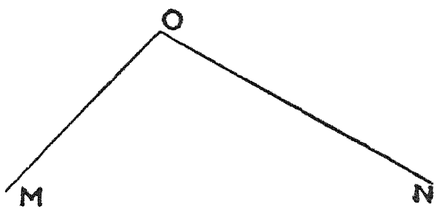
An angle greater than two right angles is a *reflex angle.*

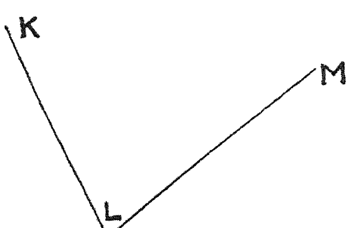
Ex. 8. Measure the following angles. What kind of angle is each?

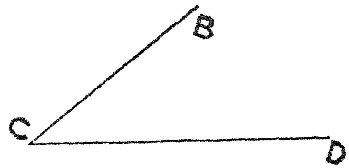


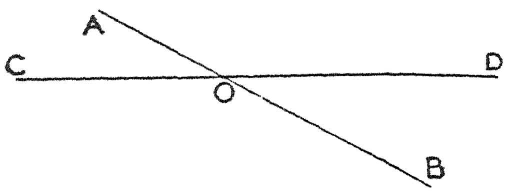

X
Y
Z


A
O
B


O
M
N


K
M
L





(In the last figure measure all the four angles).

Ex. 9. Draw a straight line BC 5·2 cm. long. With centre B and radius 3·7 cm. draw an arc. Again with centre C and radius 4 cm. draw another arc to cut the former arc at O. Join OB and OC. Measure the angles OCB, OBC and BOC. How many vertices has the figure?

Ex. 10. Draw the following angles—

(1) angle XOY = 28° (2) angle AOB = 90° (3) angle POQ = 112° (4) angle MXN = 60° (5) $\angle$XZY = 170° (6) $\angle$KLM = 159° (7) $\angle$PQR = 77° (8) $\angle$POQ = 179° (9) $\angle$RST = 56° (10) $\angle$ACB = 120°.

Ex. 11. Draw a straight line BC of length 1·7″. At B make an angle CBA equal to 60°. Make BA 1·7″ long. Join AC. Measure the angles A and C.

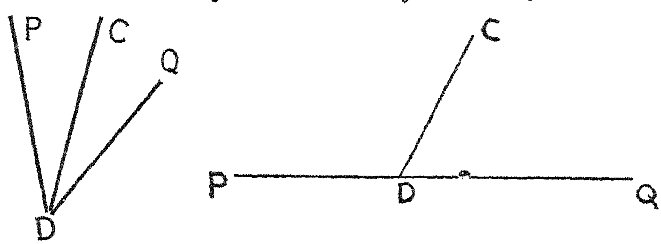
Ex. 12. Draw a straight line AB 6 cm. long. From A draw a straight line AK so that $\angle$BAK = 40°. From B draw a straight line BM making the angle ABM = 60°. Let AK and BM cross at O. Measure the angle AOB.

Ex. 13. Draw a straight line AB 7 cm. long. In AB mark any point O. At O make an angle AOC = 70°. Measure the angle BOC.

Ex. 14. AB is a straight line. At a point X in AB another straight line OX stands on it. Measure the angles OXA and OXB. What is their sum ?

Ex. 15. Draw a straight line PQ. Mark a point D in it. From D draw any straight line DC. Measure the angles CDP and CDQ. What is their sum ?

If at a point two angles are formed having a common arm they are called *adjacent angles*.



$\angle$ PDC and $\angle$ QDC are *adjacent angles*.

Ex. 16. (a) Draw several pairs of straight lines such that in each pair one straight line stands on another. Find the sum of the adjacent angles in each case.

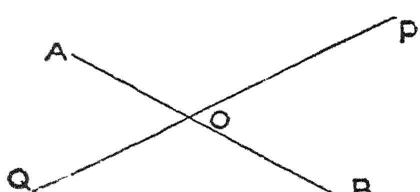
(**b**) From the results of exercises 13 to 16 what do you learn about the sum of the adjacent angles which one straight line makes with another on the same side of it ?

Ex. 17. Make an angle AOP = 37°. Produce PO to any point B. How many degrees are there in ∟AOB ? Verify by measurement.

Ex. 18. AX is a straight line standing on PQ (A being in PQ). What will be the magnitude of the other angle, if ∟PAX is equal to—(1) 30° (2) 60 (3) 90 (4) 150 (5) 75° (6) 165° (7) 87° (8) 123 (9) 147 (10) 136 ?

Ex. 19. Two straight lines AB and CD cut one another at O. If ∟AOC = 60° what is the magnitude of—(1) ∟BOC (2) ∟BOD (3) ∟AOD. What is the sum of the four angles ? Verify by measurement.

Ex. 20. Two straight lines MN and PQ cut one another at E so that ∟MEQ = 125°. What is the magnitude of each of the other angles ? Verify by measurement. What is the sum of the four angles ?



If two straight lines cross one another the angles opposite to one another are called *vertically opposite angles*. In the figure ∟AOP and ∟BOQ are vertically opposite angles. Name another pair of such angles in the figure.

Ex. 21. In exercises 19 and 20 which angles do you find equal ?

Ex. 22. Draw several pairs of straight lines such that in each pair the two straight lines cut one another. Find the magnitude of vertically opposite angles in each case. What do you notice ?

Ex. 23. What do you learn from the results of the last three exercises ?

Ex. 24. (*a*) In a circle of any radius draw a diameter AB. Mark any point P on the circumference. Join PA and PB. Measure the angle APB.

(*b*) Describe circles of different radii. Measure in each circle the angle subtended at the circumference by the diameter. What do you notice ?

Ex. 25. (*a*) Describe a semi-circle on a diameter AB equal to 6 cm. Mark points P, Q, R, S on the semi-circumference. Join them to A and B.

Measure the angles APB, AQB, ARB and ASB. What do you notice ?

(*b*) Draw several semi-circles and find the angle subtended by the diameter at the circumference in each case.

What do you learn from these results ?

Ex. 26. With centre O describe a circle of radius 4·5 cm. Mark two points A and B on the circumference. On the same side of AB mark two more points P and Q on the circumference.

Join each of the points P and Q to A and B. Measure the angles APB and AQB.

Join OA and OB and measure the angle AOB.

Ex. 27. In a circle of radius 5cm. and centre O mark an arc AB. On the circumference mark a point X on the same side of AB as O. Join each of the points O and X to A and B. Measure the angles AXB and AOB. How many times is ∟AOB of ∟AXB?

Ex. 28. Draw a circle of any radius and call the centre C. AB is any arc of it. M and N are two points on the circumference on the same side of AB. Join each of the points M, N and C to A and B. Measure the angles subtended by AB at C, M and N. Which angles are equal? How many times is the magnitude of ∟ ACB of ∟ AMB?

Ex. 29. From your results of exercises 27 and 28 what do you learn about the angles subtended by the same arc at the centre and the circumference?

Ex. 30. In a circle of any radius mark an arc. Draw any two angles so that they stand on the same arc and on the same side of it, and have their vertices on the circumference. Measure the angles. What do you notice?

CHAPTER IV.

BISECTION OF A STRAIGHT LINE.

Ex. 1. Draw a straight line AB 3" long. Mark the middle point of AB.

Ex. 2. Draw a straight line XY 4 cm. long and find its middle point.

Ex. 3. Draw a straight line PQ 1·6" long. With centre P draw a circle of radius PQ. With centre Q and with the same radius draw another circle cutting the first circle at M and N. Join MN. Let MN cut PQ at O. Measure PO and QO.

Ex. 4. Draw a straight line AB 3·3" long. Can you mark the middle point? With centre A and radius AB draw two arcs one above AB and one below it. Again with centre B and with the same radius similarly draw arcs cutting the former arcs at X and Y. Join XY. Let XY cut AB at O. Measure AO and BO. What do you notice?

Ex. 5. Draw a straight line XY of length 5·7 cm. Can you mark the middle point? With centre X and radius equal to more than half XY draw arcs on both sides of XY. Similarly with centre Y and with the same radius draw arcs cutting the former arcs at A and B. Join AB. Let AB cut XY at O. Measure OX and OY. What do you notice?

Ex. 6. Draw a straight line AB 2·3" long. With ruler and compasses and using the method in

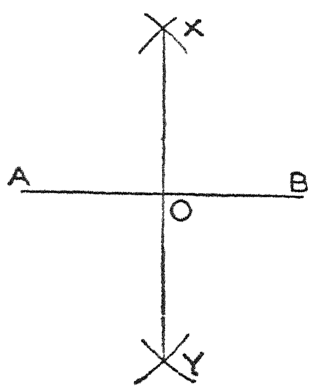
Ex. 5. find the middle point of AB. Write what all you have done.

Ex. 7. Find the middle point of a straight line 3·5' long with ruler and compasses only.

Bisect means, divide into two equal parts.

CONSTRUCTION I.

Bisect a given straight line AB.



Construction :—Draw the straight line AB.

With centre A and radius greater than half AB draw arcs on both sides of AB.

Again with centre B and with the same radius draw arcs to cut the former arcs at X and Y. Join XY. Let XY cut AB at O.

Then AB is bisected at O.

Ex. 8. Bisect a straight line MN 6·7 cm. long.

Ex. 9 Bisect a straight line AB of length 8·8 cm. Call the middle point O. Again bisect AO at P and OB at Q. Measure AP, PO, OQ and QB. Into how many equal parts have you divided AB ?

Ex. 10 Divide a straight line PQ 9 cm. long into four equal parts.

Ex. 11 Divide a straight line 11 cm. long into four equal parts.

Ex. 12. Draw a straight line AB equal to 7·5 cm. Find a fourth part of it.

Ex, 13. Take two points P and Q 7 cm. apart. Find a point A distant 4·5 cm. from P as well as from Q. Join AP, AQ and PQ. Find the middle points of AP, AQ and PQ.

Ex. 14. AB is a straight line 3·7" long, and O is its mid-point. Describe semi-circles on OA, OB and AB.

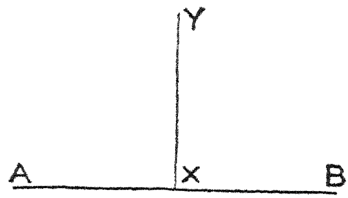
Ex. 15. Draw a straight line AB 10 cm. long. Divide it into eight equal parts.

Ex. 16. Draw a straight line AB equal to 5". Divide AB into 4 equal parts at P, Q and R. Describe semi-circles on AB, AP, PQ, QR and RB.

CHAPTER V.

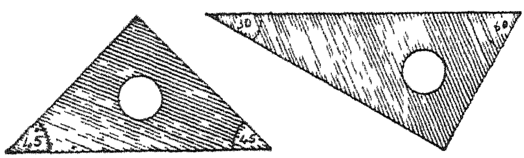
PERPENDICULARS.

If a straight line be at right angles to another straight line, it is said to be *perpendicular to* it.



XY is at right angles to AB. XY is perpendicular to AB.

Set Squares—



In your instrument box you have two flat pieces as in the figure, each with two sides set square, or forming a right angle.

One is with angles 90°, 45° and 45° at the three corners; and the other with angles 90°, 60° and 30°.

[It must be remembered that the angles at the corners of set-squares should not be relied on always for constructing angles].

Ex. 1. With the help of set squares make the following angles :—

(1) 90° (2) 45 (3) 30° (4) 60° (5) 75° (6) 150° (7) 135°.

Ex. 2. Draw a straight line AB. Mark a point X in AB. Take a set-square and place one of the sides containing the right angle along AB so that the right-angled corner is at X. Draw a line XY along the other side of the right angle. XY is perpendicular to AB.

Ex. 3. Draw a straight line PQ 8 cm. long. Mark O the middle point of PQ. Draw OR perpendicular to PQ as in ex. 2.

I—Perpendicular-bisector of a straight line.

Ex. 1 In construction I on page 28 measure the angles which the bisector makes with the straight line AB. What do you notice?

Ex. 2. Draw a straight line AB 2·5 in. long. Draw (as in construction I) XY bisecting AB at O. Measure the four angles that XY makes with AB. What kind of angles are they?

Ex. 3. Draw a straight line PQ 7 cm. long. Bisect PQ at right angles.

Ex. 4 Bisect a straight line 2·9 " long by a straight line at right angles to it. Write the construction.

Ex. 5. Draw a straight line AB 2·7" long. Bisect AB at right angles by a straight line XY at O.

[*XY is called the perpendicular-bisector of AB.*]

The line which bisects is called the bisector.

Ex. 6. Draw a straight line MN 6·5 cm, long. Draw the perpendicular-bisector of MN.

Ex. 7. Draw a straight line XY 6 cm. long. With centre X draw an arc of radius 5 cm. above the line. Again with centre Y and radius 4 cm. draw another arc to cut the former arc at P. Join PX and PY. Draw the perpendicular-bisectors of PX and PY. Let the bisectors meet in S. With centre S and radius SP describe a circle.

Ex. 8. Draw a straight line BC 2·5" long. Find a point A distant 1·5" from B and 2" from C. Join AC and AB. Draw the perpendicular-bisectors of BC and AB. Let the bisectors meet in O. With O as centre and radius OA describe a circle.

Ex. 9. Two points M and N are 5 cm. apart. Another point K is at a distance of 5 cm. from M as well as from N. Join MK and NK and draw perpendicular-bisectors of MK and NK. Let the bisectors meet in C. With C as centre and radius CK describe a circle.

Ex. 10. Two points are 2·7" apart. A third point is distant 2" from the first and 1·8" from the second. Following the method in exercises 8 and 9 describe a circle to pass through the three points. Write the construction.

II—To draw a perpendicular to a straight line at a given point in it.

Ex. 1. Draw a straight line XY 6 cm. long. Draw PQ the perpendicular-bisector of XY, bisecting it at O. OP is perpendicular to XY at O.

Ex. 2. Draw a straight line AB of length 7 cm. Mark in AB a point X 3 cm. from A. Again in AB mark two more points P and Q each 1 cm. from X. Draw MN the perpendicular bisector of PQ. Does MN pass through X? How is MX related to AB?

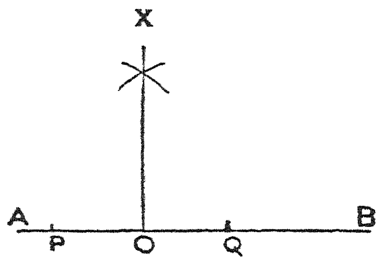
Ex. 3. Draw a straight line AB 7·7 cm. long. Mark a point O in AB 4 cm. from B. In AB mark two more points P and Q equidistant from O. Draw XY the perpendicular bisector of PQ. Does XY pass through O? Measure the angle XOA. How is XO related to AB?

Ex. 4. Draw a straight line AB 6·9 cm. in length. Mark a point P in AB at a distance of 3 cm. from A. With the help of exercises 3 and 4 draw at P a perpendicular to AB.

Ex. 5. Draw a straight line XY 3·5" long. Draw a perpendicular to XY at a point O in it, distant 1·8 in. from X.

CONSTRUCTION 2.

Draw a perpendicular to a straight line AB at a given point O in it.



Construction—With centre O and radius less than the length OA draw two arcs on both sides of O cutting AB at P and Q. With centre P and with radius greater than PO draw an arc above AB. Again with centre Q and with the same radius draw an arc to cut the former arc at X. Join XO. Then OX is perpendicular to AB.

Ex. 6. Draw a straight line PQ equal to 2·8" in length. At a point M in PQ, 1·7" from P, draw a perpendicular to PQ. Write the construction.

Ex. 7. Draw a straight line MN 7 cm. long. Find a point X in MN 3 cm. from N. At X draw a perpendicular XO to MN. Write the construction.

Ex. 8. Draw a straight line AB equal to 4 cm. in length. Mark a point P in AB such that AP is 3·8 cm. long. At P draw a perpendicular to AB.

(Hint: Produce AB to any point C. With centre P and with any radius draw an arc on both sides of P cutting AP at X and PC at Y. Then do as in construction 2.)

Ex. 9. At a point O distant 1 cm. from N in a straight line MN 7 cm. long, draw a perpendicular to MN. Write the construction.

Ex. 10. Draw a straight line AB 2·7 in. long. In AB mark two points X and Y such that X is 2·5" from A and Y is 2·2" from B. At X and Y draw perpendiculars to AB.

Ex. 11. At each end of a straight line BC 5·5 cm. long draw perpendiculars to it.

Ex. 12. At the ends of a straight line PQ 2" long draw two perpendiculars PS and QR in the same direction, each being 2" in length. Join RS. What kind of angles are formed at R and S?

III—A METHOD TO DRAW A PERPENDICULAR AT THE END OR VERY NEAR THE END OF A STRAIGHT LINE.

Draw a perpendicular to a straight line AB at A:—

With centre A draw a circle of any radius cutting AB at M. Starting from M and with the same radius step off points X and Y along the circumference. With centres X and Y and with the same radius draw arcs cutting one another at P. Join AP. PA is perpendicular to AB. Test your construction by measuring the angle PAB.

Ex. 13. Draw a straight line BC 6·4 cm. long. Find a point A distant 3·7 cm. from B and 5 cm. from C. Join AC. At A draw a perpendicular to AC.

Ex. 14. Draw a straight line PQ equal to 2.3 in. in length. At its middle point O draw a perpendicular XO. Make OX equal to OP in length. Similarly at P draw a perpendicular PY equal to OX and on the same side as OX. Join XY. Measure the four angles in the figure. (The mid-point of PQ must be marked by drawing the perpendicular bisector).

Ex. 15. AB is a straight line 4 cm. long. C is a point at a distance of 3 cm. from A and 6 cm. from B. Join AC and BC. At A and B draw AP and BQ perpendiculars to AB. Make the perpendiculars each equal to AB. Join PQ. Draw such figures on CB and AC also. The figure ABC does not lie within any of the fiigures formed by the perpendiculars.

IV—To draw a perpendicular to a straight line from a point outside the given line.

Ex. 1. Describe a circle with a point O as centre and radius equal to 3 cm. Draw a chord AB in it. Join O to D the mid-point of AB. Measure the angles ODB and ODA. What do you notice?

Ex. 2. Draw four different circles. Draw any three chords in each. Find what angle each

chord makes with the line joining the centre to its middle point. What do you learn from these results ?

In a circle the line joining the centre to the middle point of the chord is perpendicular to the chord.

Ex. 3. Draw a straight line AB 6 cm. long. Mark a point O above AB. Mark also a point P in AB near A. With centre O and radius OP draw an arc to cut AB again at Q. Construct the perpendicular-bisector of P Q. Does it pass through O ?

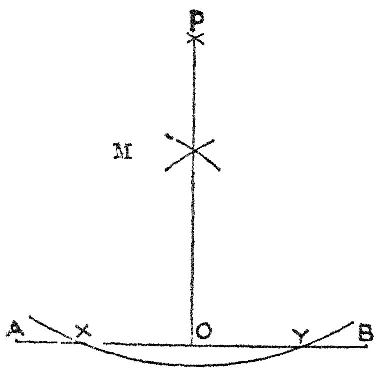
Ex. 4. Draw a straight line AB of length 3·4 in. Mark a point O outside AB. Mark also a point X in AB near B. With centre O and radius OX draw an arc cutting AB again at Y. Bisect XY at P. Join OP. Measure the angle OPA. How is OP related to AB ?

Ex. 5. Draw a straight line AB 5·3 cm. long. Mark a point P outside AB. Find two points X and Y in AB at equal distances from P. Find O the middle point of XY. Join OP. How is OP related to AB ?

Ex. 6. CD is a straight line 5·3 cm. long. O is a point distant 4 cm. from C and 3 cm. from D. In CD mark two points M and N equidistant from O. Join O to X the mid-point of MN. By measuring the angles find how OX is related to CD.

CONSTRUCTION 3.

To draw a straight line perpendicular to a given straight line from a point outside it.



Let AB be the given straight line and P the given point outside it.

Construction:—Mark a point X in AB. With centre P and radius PX draw an arc to cut AB again at Y. With centres X and Y draw arcs each of radius more than half XY. Let the arcs cut one another at M. Join PM and produce it to meet AB in O. PO is perpendicular to AB.

M should be on the same side of AB as P if there is no space for the arcs to intersect (cut) on the opposite side. Otherwise have P and M on opposite sides of AB.

Ex. 7. Draw a straight line AB 3·6 cm. long. With centre A and radius 3 cm. draw an arc above AB. With centre B and radius 5 cm. draw another arc to cut the former arc at C. From C draw a perpendicular to AB. Write the construction.

The distance of a point from a straight line is the length of the perpendicular drawn from the point to the line.

Ex. 8. Draw a straight line PQ of length 5 cm. Find a point A distant 4 cm. from P and 3·5 cm. from Q. Construct a perpendicular AO to PQ. Write the construction.

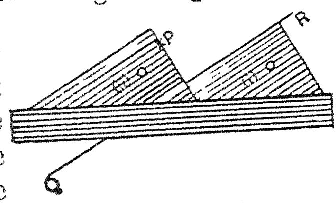
Ex. 9. AB is a straight line 2·5 in. long. P is a point distant 2·5 in. from both A and B. Join PA and PB. Draw perpendiculars from P to AB and from A to PB. Let the perpendiculars meet in O. Join BO and produce it to meet PA in X. Measure the angles PXB and AXB. How is BX related to PA?

Ex. 10. Describe a circle of radius 3 cm. Call the centre O. Draw any chord AB. From O draw a perpendicular OP to AB. Measure AP and BP. How does P divide AB?

Ex. 11. In a circle of radius 1·5" draw four chords. From the centre draw perpendiculars to the chords. By measurement find how the perpendiculars divide the chords. What do you learn?

A set square and a flat ruler can be used for drawing a perpendicular from a point P to a straight line QR.——

Place a set square so that one of the edges containing the right angle lies along the given line QR. Place the straight edge of the ruler touching the side opposite the right angle. Now hold the ruler firmly and slide the set square along it till the other edge passes through P. Then draw the perpendicular.



Ex. 12. Two points A and B are 7 cm. apart. A point P is 5 cm. from A and 4 cm. from B. Find the distance of P from the straight line AB.

Ex. 13. Draw a straight line MN 4·6 cm. long. At M and N construct perpendiculars MP and NQ each being equal to 3 cm. in length. Join PN. Find the distances of Q and M from PN. What do you notice?

Ex. 14. Two trees in a row are 300 yards apart. A third tree is 200 yards from one and 250 yards from the other. From a figure drawn to the scale 100 yds. = 1" find how far the third tree is from the line joining the first two trees.

Ex. 15. Draw a straight line AB of length 2·3 in. Mark a point X distant 1·5 in. from A and 2 in. from B. From X draw lines XP, XQ, XR and

XS to AB. From X draw also the perpendicular to AB. Of these which is the shortest line ?

Ex. 16. Draw any straight line PQ. Mark a point O above it. From O draw several lines to PQ. Draw also the perpendicular from O to PQ. Measure all the lines drawn from O to PQ. Of these which is the shortest line?

What do you learn from your results of Ex. 15 and 16 ?

The perpendicular is the shortest distance of a point from a straight line.

CHAPTER VI.

BISECTION AND CONSTRUCTION OF ANGLES

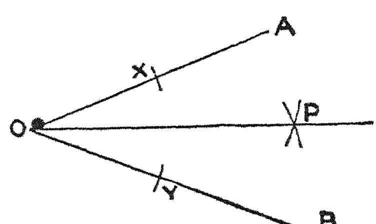
I. Bisection of Angles.

Ex. 1. Draw an angle $AOB = 55°$. With centre O and any radius draw an arc to cut the arms at P and Q. With centres P and Q and with the same radius draw arcs to cut one another at X. Measure the angles AOX and BOX.

Ex. 2. PMQ is an angle of $75°$. Mark two points K and L in MP and MQ respectively equidistant from M. Find another point N equidistant from K and L. Join MN. Measure the angles PMN and QMN. What do you notice ?

CONSTRUCTION 4.

Bisect a given angle AOB.



Construction—With vertex O as centre and with any radius draw arcs to cut the arms at X and Y.

Again with centres X and Y and with the same radius in each case draw arcs to cut one another at P. Join OP.

OP bisects the angle AOB.

[If the number of degrees in an angle be such that its half is a whole number then the bisection of the angle can be done after marking half the angle with the protractor itself].

Ex. 3. Draw a straight line AB 2" long. Find a point P distant 1 in. from A and 1·5 in. from B. Join AP. Bisect the angle PAB. Write the construction.

Ex. 4. Two points P and Q are 6 cm. apart. Find another point R distant 5 cm. from P and 4 cm. from Q. Join PR and RQ. Bisect the angle PRQ. Write the construction.

Ex. 5. Draw a straight line AB 2·3 in. long. At A and B draw in the same direction, AD and BC perpendiculars to AB. Bisect the angles DAB and ABC. Let the bisectors meet in P. Find the magnitude of the angle APB.

Ex. 6. Draw a straight line AB. From any point O in it draw a straight line OP. Draw OX and OY the bisectors of the angles AOP and BOP. Measure the angle XOY.

Ex. 7. Draw a pair of straight lines such that one straight line stands on another. Bisect the adjacent angles. Find the magnitude of the angle between the bisectors.

Ex. 8. What do you learn from the results of exercises 6 and 7 ?

II. To draw some angles with ruler and compasses only.

Ex. 9. Draw a straight line AB of any length. At A draw AP perpendicular to AB. Bisect the angle PAB by a straight line AX. How many degrees are there in the angles XAP and XAB?

Ex. 10. Draw a straight line XY. Take any point O in it. At O draw a perpendicular OP. Draw OA and OB the bisectors of the angles POX and POY respectively. Find the magnitude of (1) $\angle$YOB (2) $\angle$YOA (3) $\angle$AOB?

Ex. 11. At a point X in a straight line AB draw a perpendicular XO. Draw XP the bisector of the angle OXB. Again draw XM and XN the bisectors of the angles PXB and PXO respectively. Find the magnitude of (1) $\angle$AXO (2) $\angle$OXP (3) $\angle$OXN (4) $\angle$AXN (5) $\angle$AXP (6) $\angle$MXB (7) $\angle$OXM (8) $\angle$AXM.

Ex. 12. With ruler and compasses draw angles of the following magnitude :—

(1) 90° (2) 45° (3) $22\frac{1}{2}^\circ$ (4) $67\frac{1}{2}^\circ$ (5) 135° (6) $112\frac{1}{2}^\circ$ (7) $157\frac{1}{2}^\circ$.

Ex. 13. Draw a straight line AB 1·7 in. long. At A and B (with ruler and compasses only) make angles each equal to 45°. Let the arms other than AB meet in C. Measure the angle ACB.

Ex. 14. Draw a straight line AB 5 cm. long. From A draw a straight line AP so that $\angle$ PAB is equal to 45°. From B draw a straight line BQ so that angle ABQ = 90°. Let AP and BQ meet in O. Measure the $\angle$ AOB.

Ex. 15. AB is a straight line 2" long. At A make an angle equal to 22½°. At B make an angle equal to 45°. Let the arms other than AB meet in O. Find the distance of O from AB.

Ex. 16. With centre O describe a circle of radius 4·5 cm. On the circumference mark two points P and Q 4·5 cm. apart. Join OP and OQ. Measure the angle POQ.

Ex. 17. Describe a circle of radius 1·2 cm. with a point C as centre. Mark a point X on the circumference. With X as centre and radius equal to CX draw an arc to cut the circumference at Y. Join CX and CY. Measure the angle XCY.

Ex. 18. Draw a straight line AB. With centre A draw an arc of any radius to cut AB at C. With C as centre and radius equal to CA draw an arc to cut the former arc at D. Join AD. Measure the angle DAC.

Ex, 19. Give a construction for drawing with ruler and compasses an angle equal to 60°.

Ex. 20. PQ is a straight line. O is a point in it. OX is perpendicular to PQ at O. At O make an angle QOM = 60° (with ruler and compasses

only). OM is between OX and OQ. Draw OC and OA the bisectors of $\angle$MOX and $\angle$MOQ. Find the magnitude of (1) $\angle$QOA (2) $\angle$COM (3) $\angle$MOP (4) $\angle$QOC (5) $\angle$POC (6) $\angle$POA (7) $\angle$XOA (8) $\angle$COA.

Ex 21 With ruler and compasses draw angles of the following magnitude :—

(1) 60° (2) 30 (3) 15° (4) 75° (5) 120° (6) 150°.

Draw a separate figure for each angle.

Ex. 22. Draw a straight line AB 5·7 cm. long. At A and B make angles (with ruler and compasses only) each equal to 60°. Let the arms other than AB meet in C. Measure the angle ACB and the lengths of AC and BC.

Ex. 23. BC is a straight line 2·7 in. long. Through B draw a line BP making an angle of 60° with BC. Again through C, on the same side of BC, draw another straight line making an angle of 30 . Let these straight lines cross at A. Measure the angle BAC. With the mid-point of BC as centre and radius equal to ½BC describe a circle. (Protractor should not be used for drawing angles of 30° and 60).

III. To make an angle equal to a given angle.

Ex. 24. Mark two points A and B 7 cm. apart. Join AB. With centre A and radius 2·5 cm. draw a circle cutting AB at P. Again with

centre B and with the same radius draw a circle to cut BA at X. In the circle of centre A draw a chord PQ equal to 1 cm. Draw a chord XY equal to PQ in the other circle. Join AQ and BY. Measure the angles PAQ and XBY. What do you notice ?

Ex. 25. In two equal circles draw two equal chords one in each. Measure the angles subtended by the chords at the centre.

What do you learn from Exercises 24 and 25 ?

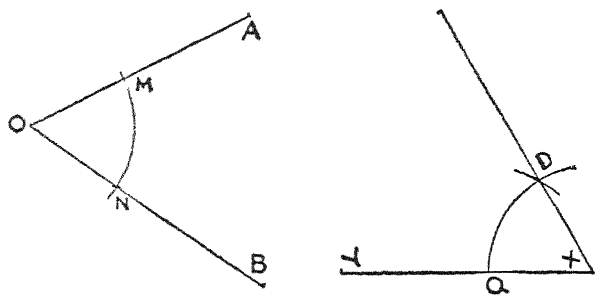
Ex. 26. Draw an angle $AOB = 75°$. Draw a straight line XY 3 cm. long. With centre O and with any radius draw a circle to cut OA at P and OB at Q.

Again with centre X draw a circle of the same radius to cut XY at M. With centre M and radius equal to PQ draw an arc to cut the circle at N. Join XN. Measure the angle NXY. Is it equal to $\angle AOB$?

Ex. 27. Make an angle $ROS = 49°$. Draw another straight line MN equal to 7 cm. With centre O and with any radius draw an arc to cut OR at X and OS at Y. With centre M and with the same radius draw a similar arc to cut MN at Q. With centre Q and radius equal to XY draw an arc to cut the former arc at P. Join PM. Measure the angle PMN. What do you notice ?

CONSTRUCTION VI.

At a point in a straight line, make an angle equal to a given angle.



AOB is the given angle. XY is another straight line.

It is required to make an angle at X equal to ∟AOB.

Construction—With centre O and any radius draw an arc to cut OA at M and OB at N. Again with centre X and with the same radius draw a similar arc to cut XY at Q. With centre Q and radius equal to MN draw an arc to cut the former arc at D. Join DX.

Angle DXY is equal to angle AOB.

Ex. 28. Draw a straight line AB = 5 cm. in length. Mark a point C distant 4 cm. from A and 3 cm. from B. Join AC. Draw another line XY equal to 6·5 cm. At X with XY make an angle equal to ∟CAB. Write the construction.

Ex. 29. Draw a straight line BC 2·1" long. Find a point O distant 1·5" from B and 1·6" from C. Join OC. At B with BC make an angle equal to $\angle$OCB.

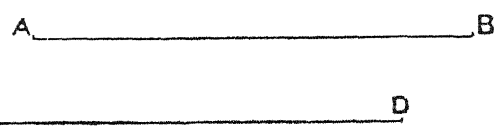
Ex. 30. Draw a straight line AB 2·5" in length. Mark a point P distant 2" from A and 1·6" from B. Join AP. In AB mark a point X 1" from B. Through X draw a straight line XQ so that P and Q are on the same side of AB and $\angle$BXQ = $\angle$PAX. Produce PA and QX both ways. Do they meet?

CHAPTER VII.

PARALLELS.

Place your scale flat on the paper. Draw two lines along the two lengthwise edges. Produce them both ways. Do they meet? These two straight lines are *parallel*. They do not meet.

Parallel straight lines do not meet when produced either way.

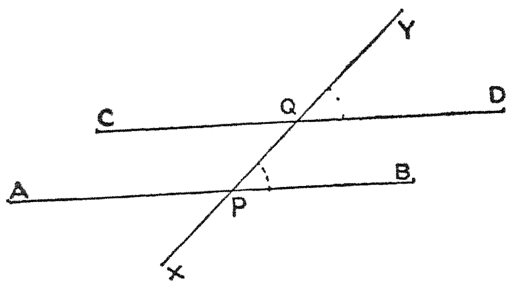


AB and CD are parallel.

The two lengthwise edges of your table are parallel. Give half a dozen examples of such parallel straight lines.

AB and CD are straight lines. A third line XY cuts them at P and Q.

The angles QPB and YQD are called *corresponding angles.*



Name other pairs of corresponding angles in the figure.

If ∟ Y Q D = ∟ Q P B, we say that Q D and P B point in the same direction.

Straight lines which have like directions are said to be parallel. Such lines will never meet, however far they may be produced.

When a straight line cuts two other straight lines, if a pair of corresponding angles are equal, then the two straight lines are parallel.

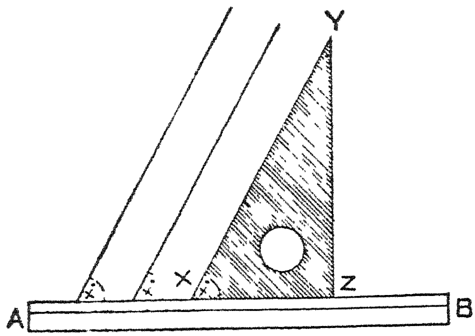
Ex. 1. AB is a straight line 5·6 cm. long. P is a point distant 4 cm. from A and 5 cm. from B. Join AP and produce it to any point M. Through P draw a straight line PQ on the same side of PA as AB, so that $\angle$MPQ = $\angle$PAB. What lines are AB and PQ called?

Ex. 2. Draw a straight line AB = 2·5 inches in length. Mark a point X distant 2" from A and 1·5" from B. Through X draw XY parallel to AB by making corresponding angles equal.

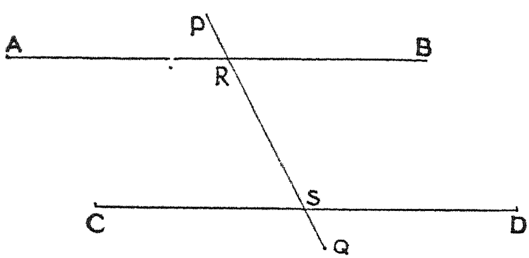
Ex. 3. Draw a straight line AB 5·5 cm. long. Mark a point P in AB. With the help of the set square, at A and P make angles BAX and BPY each equal to 45°. AX and PY are on the same side of AB. Produce AX and PY both ways. Do they meet?

Ex. 4. In a straight line PQ 7 cm. long mark two points X and Y such that the length of PX = 2 cm. and that of QY = 1·8 cm. At X make an angle PXA = 60°. At Y make an angle PYB = 60°. Produce AX and BY both ways. Do they meet?

Set squares also can be conveniently used for drawing parallel straight lines.



Place a set square XYZ with one of its edges XZ touching the straight edge of the ruler AB, placed flat on the paper. Holding the ruler firm slide the set square along AB. In various positions of the set square draw lines along XY. Since always angle X is the same, and since X in the different positions gives the magnitude of the corresponding angles, all the straight lines drawn are parallel to one another.



AB and CD are parallel. PQ is a straight line cutting AB at R and CD at S.

$\angle ARS$ and $\angle RSD$ are called *alternate angles.*

In the figure name two more alternate angles.

Measure $\angle ARS$ and $\angle RSD$. What do you notice? Measure also the angles BRS and CSR. What do you again notice?

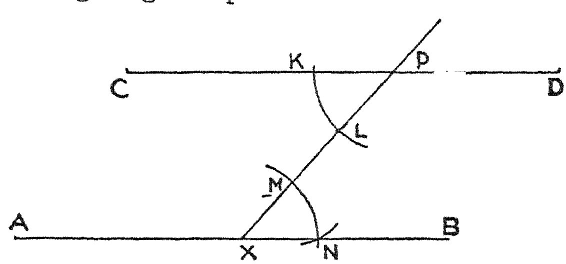
If two straight lines are parallel and a third line meets them, the alternate angles are equal.

Ex. 5. Draw a straight line AB of any length. Take a point O outside it. Through O draw a line to cut AB at P. Again through O draw another straight line OC in a direction opposite to PB, so that $\angle POC = \angle OPB$. Produce CO and AB both ways. Do they meet? What line do you call AB and CO?

Ex. 6. Draw a straight line AB 6 cm. long. Mark a point D distant 3 cm. from A and 5 cm. from B. Through D draw a straight line parallel to AB.

CONSTRUCTION 6.

Draw a straight line parallel to a given straight line through a given point.



CD is the given straight line and X the given point.

Construction—Through X draw a straight line XP meeting CD in P. With centre P and radius less than XP draw an arc to cut XP at L and PC at K.

Again with centre X and with the same radius draw an arc to cut PX at M. With centre M and radius KL draw an arc to cut the former arc at N. (PK and XN are in opposite directions). Join XN.

XN is parallel to CD.

Ex. 7. Draw a straight line AB = 2·6" in length. With centre A and radius equal to 2" draw an arc. Again with centre B and radius = 1·7 in. draw another arc to cut the former arc at C. Through C draw a straight line parallel to AB.

Ex. 8. In exercise 7 join CA. Draw through B a line parallel to CA.

Ex. 9. AB is a straight line 7 cm. long. Find a point P distant 4 cm. from A and 5 cm. from B. Through P draw a straight line PQ parallel to AB. Mark any three points X, Y and Z in PQ and from each draw perpendiculars to AB. Measure the lengths of the perpendiculars.

Ex. 10. Draw a straight line PQ. At a point O in PQ draw a perpendicular OX 6 cm. in length. Through X draw a straight line MN parallel to PQ. Mark any three points in PQ, and any three points in MN. From each point so marked drop a perpendicular to the opposite parallel. Measure the lengths of these perpendiculars. What do you notice?

Ex. 11. Draw a straight line AB 6·7 cm. in length. Mark a point O at a distance of 6 cm. from the line AB. Through O draw a parallel to AB. Measure the perpendicular distance of any point on one line from its parallel.

Ex. 12. Draw a straight line MN 2·3" long. From a point K distant 1·6" from M and 2" from N draw a straight line parallel to MN. Find the perpendicular distance between the parallels.

Ex. 13. Draw a straight line CD 6 cm. long. In CD mark two points P and Q, each 2 cm. from C and D respectively. At P draw a perpendicular PX to CD. Through Q draw QY parallel to PX. What is the distance between the parallel straight lines?

Ex. 14. Draw a straight line XY parallel to another straight line AB at a distance of 4 cm.

Angles DQP and BPQ are *interior angles* on the same side of the cutting line (fig. on page 50).

Ex. 15. In the figure on page 50 if the angle BPQ = 60° find the magnitude of the following :—

(i) $\angle$XPB (ii) $\angle$PQD (iii) $\angle$YQD (iv) $\angle$PQC (v) $\angle$APQ (vi) $\angle$APX (vii) $\angle$CQY. Which of these angles are equal?

What is the value of (1) $\angle$BPQ + $\angle$PQD (2) $\angle$APQ + $\angle$PQC.

Ex. 16. Draw a pair of parallel straight lines KL and MN, the distance between the parallels being 1". Draw another straight line cutting MN at X and KL at Y. If the angle YXN = 110°.

(i) find the magnitude of each of the other angles.

(ii) which sets of angles are equal ?

(iii) what is the value of (1) ∟NXY + ∟XYL (2) ∟MXY + ∟KYX ?

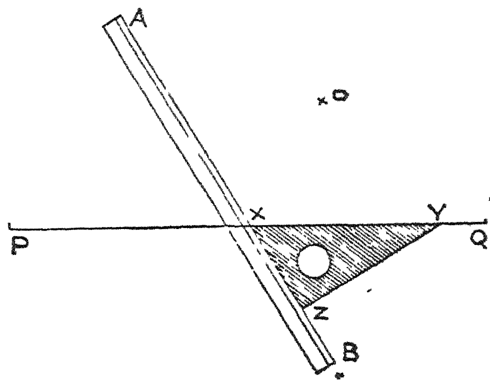
Ex. 17. If two straight lines are parallel to one another and a third line cuts them what-do you learn from the results of exercises 15 and 16 about (i) pairs of corresponding angles (ii) the sum of a pair of interior angles on the same side of the cutting line ?

Parallel lines drawn with ruler and set squere :—

Ex. 18. Draw a straight line PQ 7 cm. long. Through a point M distant 5 cm. from P and 5·3 cm. from Q, draw a straight line MN parallel to PQ. Join MP. Draw the bisectors of the angles QPM and NMP. Let the bisectors meet in O. Measure the angle MOP ?

CONSTRUCTION 7.

Through a given point (O) draw a straight line parallel to a given straight line (PQ) using a set square and ruler.



Construction—Take a set-square XYZ and place it with its longest side XY (the side opposite to the right angle) along PQ. Place also a flat ruler with its straight edge touching XZ.

Hold AB firmly and slide XZY along AB until O lies on XY. In that position draw a straight line through O along XY. This is the required straight line parallel to PQ.

Ex. 19. Draw a straight line AB 5 cm. long. Through A draw a straight line AX making an angle of 45° with AB. From A, along AB, set off lengths AP, PQ, QR, RS and SB, each equal to 1 cm. With set-square and ruler draw straight lines parallel to AX through P, Q, R, S and B.

Ex. 20. Draw a straight line PQ=5·5 in. Through P draw a straight line PA making an angle of 60 with PQ. Through Q draw QB parallel to AP in the opposite direction. From P, along PA, set off three equal lengths PK, KM and MN. On QB also with the same length set off three points X, Y and Z. Join NX, MY and KZ cutting QP in C, D and E. Measure QC, CD, DE and EP. What do you notice?

Ex. 21. Draw a straight line PQ 2·5 cm. long. Through P draw a straight line PR making any angle with PQ. Along PR step off six equal lengths PA, AB, BC, CD, DE and EF. Join FQ. Through A, B, C, D and E draw parallels to FQ meeting PQ in A_1, A_2, A_3, A_4, A_5. With the dividers compare the lengths PA_1, $A_1 A_2$, $A_2 A_3$, $A_3 A_4$, $A_4 A_5$, $A_5 Q$. What do you notice?

Ex. 22. At A in the straight line AB, 5·5 cm. long, draw a perpendicular AP. In AP mark four equal lengths AX, XY, YZ and ZW. Join WB. Through X, Y and Z draw parallels to WB meeting AB in K, M and N. Compare the lengths AK, KM, MN and NB. What do you notice?

Ex. 23. Draw a straight line AB 2·7" long. Use the method in ex. 20 to divide AB into five equal parts.

Ex. 24. Divide a straight line XY 5 cm. long into three equal parts.

CHAPTER VIII.

DIVISION OF STRAIGHT LINES.

Ex. 1. Draw a straight line AB 3" long. Keeping the arms of the dividers 1" apart step it off along AB, beginning from A. Into how many equal parts is AB divided?

Ex. 2. Divide with the help of dividers a straight line 4·8 inches long into four equal parts.

Ex. 3. Divide a straight line AB 5" long. into five equal parts. Call the points of division C, D, E and F. What is the value of the following ratios: (i) AC : CB (ii) AD : DB (iii) CD : DF (iv) AC : AB (v) AD : AB.

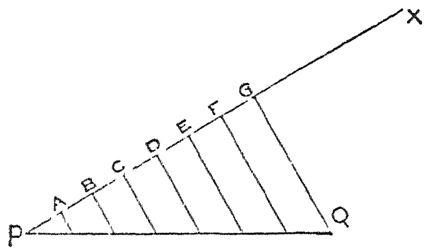
Ex. 4. Draw a straight line PQ 6 cm long Through P draw a straight line PX making any angle with PQ. Take any length between the steel points of the dividers and beginning from P step it off along PX four times. Call the points of division A, B, C, D. Join DQ. Through A, B, C draw parallels to DQ meeting PQ in R, S, T. With the dividers compare the lengths of PR, RS, ST and TQ. What do you notice?

Ex. 5. Draw a straight line AB=2.8" long and divide this into five equal parts as in Ex. 4.

CONSTRUCTION 8.

Divide a given straight line into seven equal parts.

PQ is the given straight line It is to be divided into seven equal parts.



Construction:—Draw a straight line PX making any angle with PQ. From PX mark off seven equal lengths, PA, AB, BC, CD, DE, EF, FG. Join GQ.

Through A, B, C, D, E and F draw with set square and ruler parallels to GQ to meet PQ. These parallels divide PQ into seven equal parts.

Ex. 6. Divide a straight line 7 cm. long into six equal parts. Write the construction.

Ex. 7. Divide the straight lines as directed:-

(i) AB 5·8 cm. long into three equal parts.

(ii) CD 1·5 " long into four ,, ,,

(iii) MN 2·2" long into five ,, ,,

(iv) XY 7 cm. long into eight ,, ,,

(v) RS 3·9 " long into nine ,, ,,

Write the construction in each case.

Ex. 8. Draw a straight line XY 1·6." long. From it cut off a fifth part. Write the construction.

Ex. 9. From a straight line 5·6 cm. long cut off a seventh part.

Ex. 10. Draw a straight line AB = 5 in. in length. From it cut off a part equal to three-sevenths of AB.

Ex. 11. Draw a straight line AB 3·2 in. long. Divide it into five equal parts. Call the second point of division X. What is the ratio of the parts contained in AX and XB ?

Ex. 12. Divide a straight line KL 8 cm. long into seven equal parts. Call the fourth point of division P. What is the value of the ratio of the lengths of KP and PL ?

Ex. 13. Divide a straight line PQ 2·9 " long into three equal parts. Call the second point of division X. What is the value of PX : XQ ?

Ex. 14. Divide a straight line AB 7 cm. long into five equal parts. Call the points of division A_1, A_2, A_3, A_4. What is the value of (i) $AA_1 : A_1B$ (ii) $AA_2 : A_2B$ (iii) $AA_3 : A_3B$ (iv) $AA_4 : AB$.

Ex. 15. Draw a straight line AB 7·5 cm. long. Divide AB in the ratio of 1 : 3.

Ex. 16. In a straight line XY = 2·9 " in length, find a point P which divides it in the ratio of 2 : 3.

Ex. 17. Divide a straight line :—

(i)	4 cm. long in the ratio of		1 : 2
(ii)	5 cm.	„ „ „	2 : 3
(iii)	6 cm.	„ „ „	3 : 2
(iv)	2·5 "	„ „ „	3 : 4
(v)	2·1 "	„ „ „	1 : 4

Ex. 18. Draw a straight line BC 2·7" long. Find a point A distant 3 " from B and 1·4 " from C. Join AB and AC. Divide AB at X in the ratio of 3 : 2, and AC at Y in the same ratio. Join XY. Measure ∟ AXY and ∟ ABC. Produce BC and XY both ways. What lines are BC and XY called ?

CHAPTER IX

DIRECTIONS.

If you stand facing the Sun in the morning the direction in which the Sun rises is called East (E); directly opposite to it where the Sun sets is West (W). The direction to the left of you is North (N) and to the right South (S).

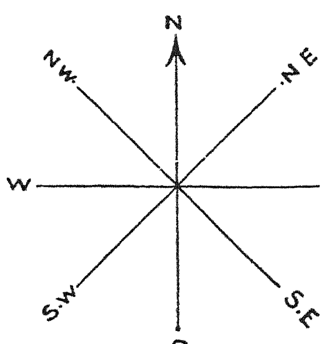
The direction mid-way between

(i) North and East is North-East (N. E.)

(ii) South and East is South-East (S. E.)

(iii) North and West is North-West (N.W.)

(iv) South and West is South-West (S. W.)



The line showing the direction North to South is at right angles to the line showing the direction East to West.

The direction N. E. is at 45° to North or East. So on for N. W., S. W., S. E.

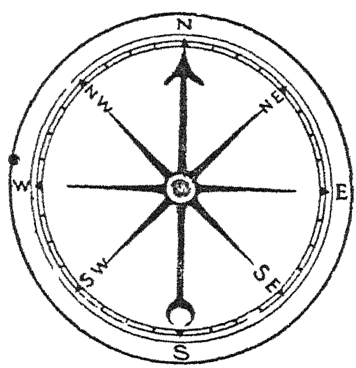
Ex. 1. Name places to the North, East, South and West of your school.

Ex. 2. Name some places to the (1) South-East (2) North-West (3) North-East (4) South-West of your school.

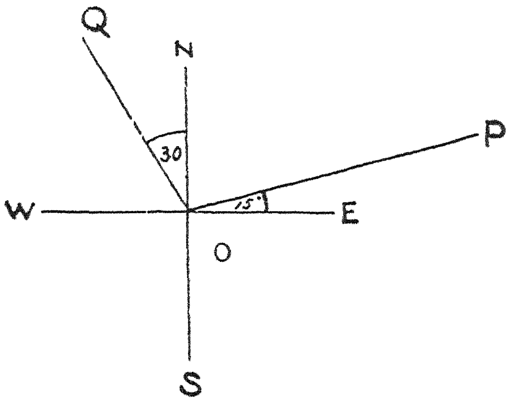
If the direction of some object from your place is between North and East at an angle of 40° with the easterly direction the bearing of the object reads 40° N. of E. from your place.

Directions can usually be determined from a Mariner's compass. In it the compass needle lies always in the direction of North and South. The direction at right angles to that shown by the compass needle is East and West.

Teach boys the use of this instrument.



THE MARINER'S COMPASS.



In the figure P is 15° North of East (E. 15 N.) of O; and Q is 30° West of North (N. 30° W.) of O.

In other words, P bears 15° North of East and Q 30° West of North as viewed from O.

Ex. 3. With a separate figure in each case explain the following directions:—

(1) N. W. (ii) N. E. (iii) S. E. (iv) S. W. (v) 30° N. of W. (vi) 30° E. of N. (vii) 25° S. of E. (viii) 37° S. of W. (ix) N. 34° E. (x) W. 20° S. (xi) S. 33° E. (xii) W. 15° S.

Ex. 4. A house bears N. E. of your School and is distant 4 miles from it. Draw a plan to show the position of the house when viewed from the school.

Ex. 5. The hospital is 20° South of West of your friend's house and is distant 400 yards. Draw a diagram to show their positions (1" = 200 yards.)

Ex. 6. George's house bears 40° W. of N of the School and is 500 yards distant. Rama's house bears N. E. of your school and is 650 yards from it. Draw a plan to show their relative positions (1 cm.=100 yards).

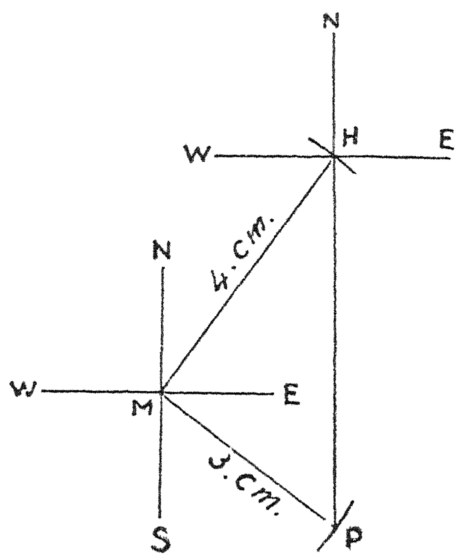
Example :—

H is a place 54° N. of E. of M and another place P is E. 36 S. of M. H and P are 4 miles and 3 miles respectively from M.

(i) Draw a plan to show their relative positions.

(ii) Find the distance of P from H.

(iii) How does P bear from H ?



First draw NS and EW cutting one another at right angles at M.

From M draw a straight line MH 4 cm. long making the angle EMH = 54°.

Again from M draw MP 3 cm. long making the angle EMP = 36°.

Join PH. The length of PH gives the distance from P to H.

To find the bearing of P from H draw through H straight lines parallel to NS and EW. The angle which HP makes with the parallel to NS gives the bearing of P from H. Here P is South of H.

Choose any suitable scale yourself when it is not given.

Ex. 7. A place X is 30 miles East of your house. Another place Y is 30 miles South of the house. What is the distance and bearing of X from Y? (Scale 10 miles = 1")

Ex. 8. One village is 4 miles to the North of your school and another 3 miles to the West. Draw a plan to the scale of 1 mile to 2·5 cm. and find the distance between the two villages.

Ex. 9. Hyderabad M. G. station is 3 miles to the South of Secunderabad, and Hussain Sagar is 3 miles to the West of Secunderabad. What is the length of the straight road from Hussain Sagar to Hyderabad M. G.? How does Hyderabad M. G. bear from Hussain Sagar?

Ex. 10. Yokohama is 4,500 miles West of San Francisco. Vancouver is 750 miles North of San Francisco. How far is Vancouver from Yokohama and how does it bear from the latter? (Scale 500 miles = 1 cm.)

Ex. 11. There are two villages. One is to the North East of your school and 3,500 yards from it. Another is S. E. of the school and 2,500 yards from it. Draw a plan and find the distance and bearing of the first village from the second. (Scale 500 yards = 1 cm.).

Ex. 12. From the harbour I view two ships, one in a North-easterly direction at a distance of 2 miles, and another in a North-westerly direction at the same distance. Find the distance between the two ships and the bearing of the second ship as viewed from the first.

Ex. 13. You get down at the Railway station of a certain place and you are told that the hospital is South East of the station at a distance of 4 miles; and the school is South West of the station at a distance of 3 miles. If you first go to the school, then from there to the hospital and return straight to the station, what distance in total do you walk? How does the station bear from the school?

Ex. 14. A is 2,400 yards North East of O. B is 3,000 yards North West of O. How far is A from B? How does it bear when viewed from B?

Ex. 15. A place P is 5 miles North East of M. Another place Q is 7 miles to the North of M. How far is P from Q? How does P bear when viewed from Q?

Ex. 16. Your friend views in a North-westerly direction from his house a fort distant 800 yards. He also sees a temple distant 1,000 yards due South. How far is the fort from the temple? How does the temple bear from the fort?

Ex. 17. The railway station bears 45° W. of N. from the School and the market bears 45° West of South from the same school. From the school the station is 1,500 yards and the market 2,500 yards. Draw a plan and find the distance between the station and the market. How does the market bear from the railway station?

Ex. 18. The hospital in 20° W. of N of your school and distant 2 furlongs. The Railway station is 40° S. of E. of the hospital distant 1½ furlongs from the same school. From a diagram find the distance and bearing of the station from the school. (110 yards = 1 ")

Ex. 19. Secunderabad bears 10° W. of S of Bolarum and is 6 miles away. Hyderabad (B. G.) bears 38° W. of S. of Secunderabad and is 4 miles from it. Find the distance and bearing of Hyderabad (B. G.) from Bolarum?

Ex. 20. Secunderabad is 30° S. of W. of Kazipet and is 80 miles from it. Dornakal is 40° S. of E of Kazipet and is 60 miles from it. How does Dornakal bear when viewed from Secunderabad? What is the distance between the two places?

Ex. 21. Secunderabad market is 700 yards from the station and bears W. 15° N. of it. The clock tower is N. 15° W. and distant 800 yards from the station. How far is the tower from the market? How does the tower bear from the market?

Ex. 22. Bidar is N. 34° W. of Vikarabad and distant 57 miles. Hyderabad is 45 miles East of Vikarabad. How far is Bidar from Hyderabad and how does it bear?

Ex. 23. Raichur bears S. W. of Secunderabad and is 120 miles distant. Wadi bears 13° S. of W. of Secunderabad and is 120 miles distant. How far is Raichur and how does it bear from Wadi?

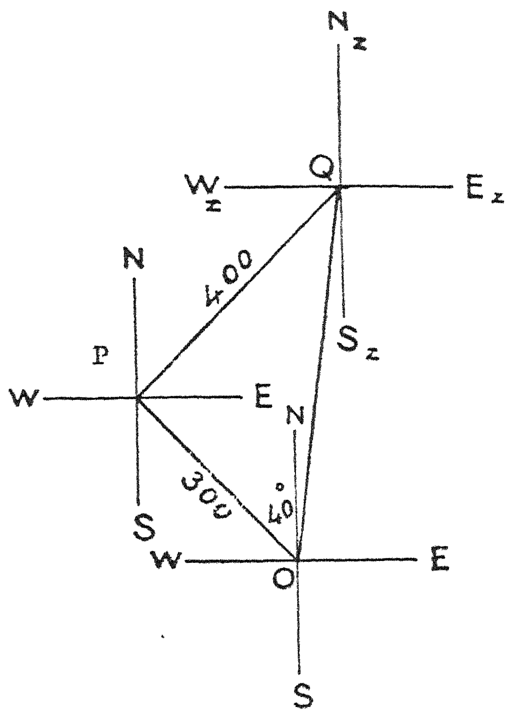
Ex. 24. Calcutta is E. 50° N. of Madras and is 800 nautical miles distant. Penang is E. 20° S. of Madras and is 1,300 nautical miles away. How does Penang bear from Calcutta and what is the distance between the two places?

Ex. 25. From O three places P, Q and R are 700 yards, 600 yards and 850 yards respectively. They bear N. 25° E., E. 30° S. and S. W. when viewed from O. How far are P and Q from R? How does each bear from R?

Example :—

From the school I walk 300 yards due N. 40° W. Then I walk 400 yards due N. E. How far am I from the school?

What is the bearing of the school from my last position ?



O is the starting place. P is the place I reach first and Q is the last position. The distance from Q to the school is the distance required and $\angle S_2QO$ is the bearing of O from Q.

Ex. 26. You walk N. W. of your house for 3 miles. Then you turn and walk $4\frac{1}{2}$ miles due East of your second position. Draw a plan of your course and find how far you are from the house.

Ex. 27. A man walks 4 miles due West. He then turns N. W. and walks 3 miles. How far is he from the starting point and what is the bearing?

Ex. 28. A boy travels 5 miles due North. He then turns N. E. and walks 12 miles. Find the distance and bearing of his starting place from his last position?

Ex. 29. From your school you walk 150 yards due East, then 300 yards due North and finally 450 yards due West. How far are you from the school? How does your last position bear from the school?

Ex. 30. Ramgopal's statue is 500 yards to the North of James Street Clock-Tower. Wesleyan High School is 150 yards to the West of the statue. How does the tower bear from the school and how far is it?

Ex. 31. Two places P and Q are 700 yards apart, Q being North West of P. Another place X bears S. E. and is 550 yards from Q. Find the bearing and distance of P from X.

Ex. 32. You start from your school and go home. Finding the weather pleasant you take a stroll. You walk 600 yards due East, then turn to your left at an angle of 30° and walk 450 yards more and reach home. How much more distant is your course than the usual one?

Ex. 33. A boy walks 3 miles due North East. Then he turns 45 to his right and walks 6 miles. Find the distance and bearing of his last position as seen from his starting place.

Ex. 34. I start from St. Mary's School and walk 200 yards due North. Then I walk 150 yards due East, and finally I turn and walk 500 yards due South. Find the distance and bearing of my last position from the school.

Ex. 35. From Vivakaverdhani school gate teacher starts and walks 200 yards to the east. Then he turns due South and walks 500 yards. Finally he turns to the West and walks 300 yards. Find the bearing and distance of the school from his final position.

Ex. 36. Two boys A and B start from your school on bicycles. A goes at 10 miles an hour North West and B goes at 12 miles an hour due W 40° S. How far will they be after half an hour and how will A bear from B.? (Scale 1 mile= 2 cm.).

Ex. 37. Four scouts A, B, C, D start from your school. A walks for half an hour due N. E. at 4 miles an hour. B walks the same distance due S. E. C goes for three quarters of an hour at 6 miles an hour due S. W. D travels due North the same distance as C. Draw a plan to show the relative positions. Find the bearing of D from B.

Ex. 38. To go to the next village I walk due North East for half an hour at 4 miles an hour; then I turn due West and walk for 45 minutes. Lastly I run due N. W. for 15 minutes at 8 miles an hour. Find the distance and bearing of the village from my place.

Ex. 39. Two cyclists agree to bike at the rate of 15 miles an hour. They start from your school. One goes due East for half an hour and then turns S. W. and goes for 20 minutes. The other bikes due East for 20 minutes, and then turns and goes N. E. for 12 minutes. Find the distance and bearing of the first cyclist from the second.

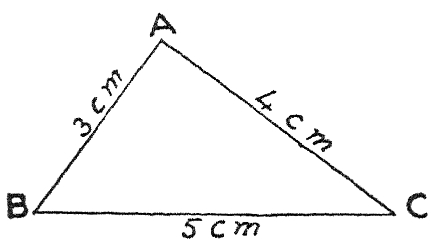
Ex. 40. A is 20 miles East of B. C is 5 miles N. E. of A. D is 10 miles South of C. E is 15 miles South West of D. Draw a plan to the scale 2·5 miles to 1 cm. to show their relative positions. If one starts from A and walks to C, D, E and B in order, what will be the total distance covered? How does E bear from D?

CHAPTER X.

TRIANGLES.

A figure bounded by three straight lines is called *a triangle.*

Draw a straight line BC 5 cm. long. Find a point A distant 3 cm. from B and 4 cm. from C. Join AB and AC.



The figure ABC is a *triangle.*

AB, BC, CA are the *sides of the triangle.*

The points A, B and C are the *vertices.*

The triangle has three angles A, B and C.

(A *triangle* is a figure of *three angles*).

The sides are represented by small letters, and the angles by capital letters.

Side BC can be written as side *a.*

How may the other sides be written?

BC is usually called the *base* of the triangle.

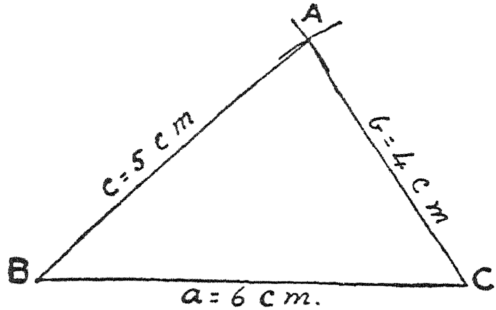
The sign for a triangle is △.

△ABC stands for triangle ABC.

I. CONSTRUCTION IX.

Describe a triangle having given the lengths of three sides.

Let the sides be $a=6$ cm , $b=4$ cm., $c=5$ cm.



Construction—Draw a straight line BC 6 cm. long. With centre B and radius 5 cm. draw an arc. Again with centre C and radius 4 cm. draw another arc to cut the former arc at A. Join AB and AC.

ABC is the required triangle.

Ex. 1. Describe triangles having been given :—

(i) $a=2''$, $b=1''$, $c=1{\cdot}8''$.

(ii) $a=1{\cdot}5''$, $b=1{\cdot}6''$, $c=2{\cdot}1''$.

(iii) $a=6{\cdot}7$ cm., $b=4{\cdot}5$ cm., $c=3{\cdot}8$ cm.

(iv) two equal sides, each equal to 7 cm. and the third side $=5$ cm.

(v) each of the sides $=1{\cdot}5$ in.

Ex. 2. Measure the angles in each of the triangles in Ex. 1 and find the sum of the three angles in each case. What do you learn about the sum of the three angles of any triangle?

Ex. 3. Two places B and C are 700 yards apart. A third place A is 600 yards from B and 400 yards from C. Draw a plan of the figure.

Ex. 4. A place P is $7\frac{1}{2}$ miles from Q. Another place R is 10 miles from P and 8 miles from Q. Draw a plan and find how far R is from the straight road PQ.

Ex. 5. The distance between my house and school is 370 yards. The police station is 500 yards from my house. The school is 340 yards from the police station. Draw a plan and find the distance of the school from the straight road joining the station and the house.

Ex. 6. Two places X and Y are 6 miles apart. A place M is 7 miles from X and 4 miles from Y. A fourth place N, on the same side of XY as M, is $3\frac{1}{2}$ miles from X and 7 miles from Y. Draw a plan and find how far N is from M.

Ex. 7. In each of the triangles in exercise 1, find the sum of any two sides. Find also whether that sum is greater or less than the length of the third side.

Ex. 8. Try to draw triangles in which

(i) $a=3''$, $b=1''$, $c=1\cdot5''$.

(ii) $a=5\cdot5$ cm., $b=2\cdot5$ cm., $c=2$ cm.

(iii) $a=4$ cm., $b=10$ cm., $c=4\cdot5''$.

(iv) $a=2\cdot1''$, $b=1\cdot2''$, $c=3\cdot5''$.

(v) $a=5$ cm., $b=3$ cm., $c=3\cdot5''$.

(vi) $a=2''$, $b=1''$, $c=3''$.

(vii) $a=7$ cm., $b=4$ cm., $c=3$ cm.

(viii) $a=1.5''$, $b=3\cdot2''$, $c=4\cdot7''$.

When do the constructions fail and why ?

Ex. 9. What is the condition that a triangle may be drawn with three given lengths as sides?

Ex. 10. Lengths of three straight lines are given as follows:

(i), 3, 4, 5. (ii) 7, 2, 5. (iii) 5, 5, 10. (iv) 6, 11, 3. (v) 12, 19, 40.

Determine, without actually drawing the figures, when all triangles can be formed with the given lengths as sides.

Triangles can be divided into three classes with regards to their sides.

(i) a $\triangle$ is *scalene* when no two of its sides are equal.

(ii) a △ is *isosceles* when any two sides are equal. The point where the equal sides meet is usually called the *vertex*, and the side opposite to it is called the *base*.

(iii) a △ is *equilateral* when all its sides are equal.

II ISOSCELES TRIANGLES

Ex. 11. Construct an isosceles triangle ABC in which AB=AC=6 cm., and BC=5 cm. Measure the angles opposite to the equal sides. What do you notice ?

Ex. 12. Construct the following isosceles triangles :

Base=	4 cm.	2·6"	1·8"	3·6cm.	5·9cm.	1·5"
Each of the equal sides=	5 cm.	1·5"	1·2"	2·8cm.	4 cm.	1·5"

In each of the triangles measure the angles opposite to the equal sides. What do you learn from these about the base angles of any isosceles triangle ?

Ex. 13. Construct an isosceles triangle ABC in which BC=6·8 cm. ; AB=AC=5·2 cm. Bisect BC at O. Join AO. Measure the following angles: ∟BAO, ∟CAO, ∟AOB and ∟AOC. What do you notice ?

Ex. 14. Construct half a dozen isosceles triangles. Join the vertex to the middle point of the base in each triangle. Cut out each triangle and fold it about the line joining the vertex to the mid-point of the base. From these you will learn that

(i) *the angles at the base are equal.*

(ii) *the line joining the vertex to the mid-point of the base bisects the vertical angle and is perpendicular to the base.*

Verify these by measurement.

Ex.. 15. Draw half a dozen more isosceles triangles. Draw the bisector of the vertical angle of each. Cut out the triangles and fold each triangle about the bisector. From these you will learn that

(i) *the angles at the base are equal.*

(ii) *the bisector of the vertical angle bisects the base and is perpendicular to it.*

Verify these by measurement.

Ex. 16. Draw a straight line BC 7 cm. long. At O the mid-point of BC draw a perpendicular. Mark any point A on the perpendicular. Join AB and AC, and measure them. What do you notice?

Ex. 17. Draw a straight line MN 2·9" long. At X, the mid-point of MN, draw a perpendicular. Mark three points A, B and C on the perpendicular. Join each point to M and N. Measure AM and AN, BM and BN, CM and CN. From these

results what do you learn about the distances of any point on the perpendicular-bisector of a straight line from its ends ?

Ex. 18. In Ex. 17, what kinds of triangles are AMN, BMN and CMN ?

Ex. 19. Construct isosceles triangles having been given—

Base =	6 cm.	5·8 cm.	2"	3·3"	7·8 cm.
The perpendicular from the vertex to the base =	4 cm.	3·7 cm.	1·2"	2"	6 cm.

Ex. 20. What is the value of each base angle of an isosceles triangle if the vertical angle is (i) 40° (ii) 90° (iii) 60° (iv) 75 (v) 125° (vi) 100° (vii) 120° (viii) half a right angle.

III—EQUILATERAL TRIANGLES

In an equilateral triangle, since all the sides are equal, the properties of an isosceles triangle hold good.

Ex. 21. What is the magnitude of each angle of an equilateral triangle ?

Ex. 22. Construct equilateral triangles on sides 3 cm., 5·5 cm., 2", 1·7", 6 cm., 2·3".

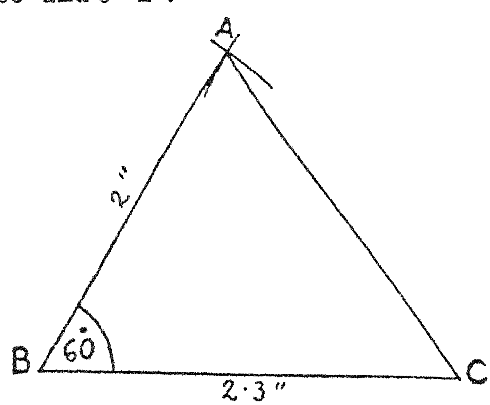
Ex. 23. Draw an equilateral triangle ABC having each side equal to 4·5 cm. Mark P, Q and R the mid-points of the sides a, b and c respectively. Join PQ, PR and RQ. By measuring the sides of the $\triangle$ PQR determine what kind of triangle it is.

Ex. 24. Construct six equilateral triangles. Join the mid-points of the sides in order, in each triangle. In each case after measuring the sides of the triangle within, determine what kind of triangle it is.

Ex. 25. Draw an equilateral $\triangle$ ABC, each side being 4 cm. long. Through A, B and C draw parallels to the opposite sides. Let these lines cut one another and form the $\triangle$ PQR. Measure the sides of this triangle and determine what kind it is.

IV—CONSTRUCTION X.

Draw a triangle ABC having given $a = 2{\cdot}3''$, $\angle B = 60^\circ$ and $c = 2''$.

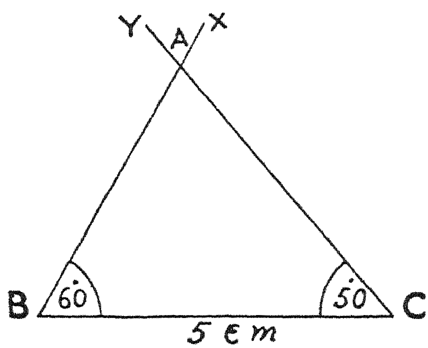


Construction—Draw a straight line BC 2·3" long. At B make an angle CBX=60°. With centre B and radius=2" draw an arc to cut BX at A. Join AC.

ABC is the required triangle.

CONSTRUCTION XI.

Draw a triangle ABC having been given a=5 cm., $\angle$B=50° and $\angle$C=50



Construction—Draw a straight line BC 5 cm long. From B draw a straight line BX making an angle of 60° with BC. Similarly from C draw a straight line CY making on the same side an angle of 50°. Let BX and CY cross at A. ABC is the required triangle.

Ex. 26. Construct the following triangles :

(i) $a = 4$ cm., $\angle B = 70^\circ$, $c = 3$ cm.

(ii) $a = 2{\cdot}6''$, $\angle B = 58^\circ$, $c = 1{\cdot}5''$.

(iii) $a = 2''$, $b = 2{\cdot}4''$, $\angle C = 105^\circ$.

(iv) $a = 3{\cdot}5$ cm., $b = 5$ cm., $\angle C = 90^\circ$

(v) $\angle A = 60^\circ$, $b = 6$ cm., $c = 7$ cm.

(iv) $\angle A = 100^\circ$, $b = 1{\cdot}8''$, $c = 2{\cdot}1''$.

Ex. 27. Construct a triangle ABC in which angle $C = 60^\circ$, $b = 5{\cdot}4$ cm. and $a = 7$ cm. (No protractor should be used). Find the distance of A from BC.

Ex. 28. Draw an isosceles triangle in which the vertical angle is 50°, and the equal sides each equal to $1{\cdot}5''$. Calculate the base angle and verify by measurement.

Ex. 29. Draw the following isosceles triangles :

Vertical angle=	45°	90°	125°	30°	100°
Each equal side=	$2''$	6 cm.	5 cm.	$1{\cdot}8''$	7 cm.

In each case find the distance of the vertex from the base.

Ex. 30. Construct the following triangles :—

(i) $a=6$ cm., $\angle B=80°$, $\angle C=40°$.

(ii) $a=1·7''$, $\angle B=110°$, $\angle C=30°$.

(iii) $\angle A=50°$, $\angle B=30°$, $c=5$ cm.

(iv) $\angle A=77°$, $\angle B=40°$, $c=3''$.

(v) $\angle A=100°$, $b=6·7$ cm., $\angle C=45°$.

(vi) $\angle A=85°$, $b=2·3''$, $\angle C=35°$.

In each of the triangles calculate the third angle, and verify by measurement.

Ex. 31. Construct a triangle ABC in which $\angle A=50°$, $\angle B=60°$, $\angle C=70°$. How many figures are possible ?

Ex. 32. Construct the following triangles.

(i) $b=5·5$ cm., $\angle A=50°$, $\angle B=110°$.

(ii) $c=3·5$ cm., $\angle C=45°$, $\angle A=95°$.

(iii) $b=2·2''$, $\angle B=50°$, $\angle C=60°$.

(iv) $a=2·5''$, $\angle B=90°$, $\angle A=30°$.

(v) $a=6$ cm., $\angle A=76°$, $\angle C=34°$.

(vi) $c=1·8''$, $\angle B=105°$, $\angle C=35°$.

(Hint : Calculate the third angle and do as in exercise 30.)

Ex. 33. Construct the following isosceles triangles :—

Base=	5 cm.	2"	9 cm.	3"	4·5 cm.
Vertical angle=	50°	70°	90°	100°	60°

Ex. 34. Try to draw the following triangles:—

(i) $a = 4$ cm., $\angle B = 150°$, $\angle C = 30°$.

(ii) $b = 1·7''$, $\angle B = 60°$, $\angle C = 120°$.

(iii) $c = 7$ cm., $\angle A = 55°$, $\angle B = 125°$.

(iv) $b = 2·5''$, $\angle B = 90°$, $\angle C = 90°$.

What difficulty does arise in each case? Give reasons.

With respect to the angles also triangles can be divided into three classes.

A triangle is said to be

(i) *right-angled* when one of its angles is a right angle.

(ii) *obtuse-angled* when one of its angles is obtuse.

(iii) *acute angled* when all its angles are acute.

Ex. 35. In a triangle.

(i) how many right angles can there be ?

(ii) „ „ obtuse „ „ „ „ ?

Give reasons.

Ex. 36. Two of the angles of a triangle are given in each set ; find the third angle and determine the nature of the triangle. (i) 30°, 40°. (ii) 50°, 80°. (iii) 60°, 30°. (iv) 45°, 45°. (v) 20°, 70°. (vi) 20°, 65°. (vii) 37°, 86°. (viii) 45°, 40°. (ix) 48°, 27°. (x) each of the base angles = 39°. (xi) the base angles together = 120°. (xii) the sum of any two angles = 87°.

V—RIGHT-ANGLED TRIANGLES

Ex. 37. Describe a semi-circle on a diameter AB of any length. Mark points X, P, Y, and Q on the arc. Join each point to the ends of the diameter. Measure the angles AXB, AYB, APB and AQB. What kind of angles are they ?

Ex. 38. Draw several circles and find in each the magnitude of the angle subtended by the diameter at the circumference. What do you notice ?

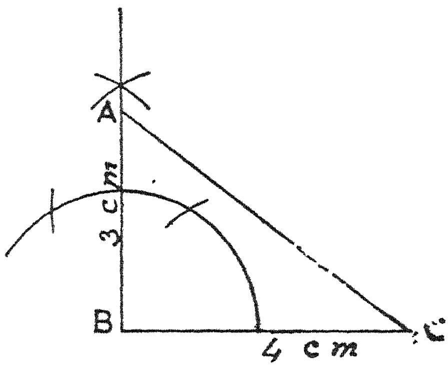
All angles in a semi-circle are right angles.

In a right-angled triangle the side opposite to the right angle is called the *hypotenuse*.

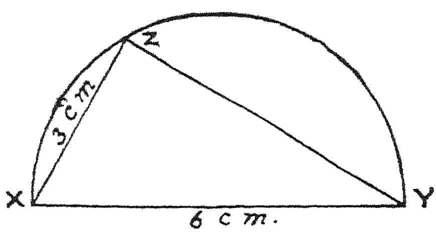
CONSTRUCTION XII.

Construct a right-angled triangle having given (i) two sides containing the right angle to be 4 cm. and 3 cm.

(ii) the hypotenuse = 6 cm. and one side = 3 cm.



Constructions—(i) Draw a straight line BC 4 cm. long. At B draw a perpendicular AB = 3 cm. Join AC. ABC is the required right-angled triangle.



(ii) Draw XY = 6 cm, the length of the hypotenuse. On XY describe a semi-circle. On the semi-circumference find a point Z 3 cm. from X. Join XZ and YZ. XYZ is the required $\triangle$.

Ex. 39. Construct (with ruler and compasses only) right-angled triangles in which the two sides containing the right angle are equal to (i) 1 cm., 8 cm., (ii) 2·8", 1·7" (iii) 5 cm., 6·7 cm., (iv) 1·4", 1·9", (v) 1·5 cm., 7 cm., (vi) 2" and 1·5".

Ex. 40. In each of the △s in exercise 39 compare the length of the line joining the mid-point of the hypotenuse to the vertex opposite to it with half the hypotenuse. What do you notice ?

Ex. 41. Construct only with ruler and compasses the following right-angled triangles.

Hypotenuse=	5 cm.	2·5"	1·8"	6cm.	3"	7.6cm
One side=	3 cm.	1"	1·2"	4·8cm.	1·5"	5·5cm

In each triangle find the length of the line joining the mid-point of the hypotenuse to the right angle. How much is it of the hypotenuse ? What do you learn ?

Ex. 42. BC is a straight line 8 cm. long. Find a point P distant 5 cm. from B, at which BC subtends a right angle. How far is P from BC ?

Ex. 43. Two points X and Y are 2" apart. Find a point O distant 1·5 " from Y at which XY subtends a right angle. What is the distance of O from XY ?

Ex. 44. A ladder when placed at a distance of 10 feet from the foot of the wall reaches a window 20 feet high. Draw a figure and find the length of the ladder. (Scale 5 feet = 1 cm.)

Ex. 45. A tent post 12 feet high is supported by ropes tied to its top and to pegs on the ground, each peg being at a distance of 10 feet from the post. What is the length of each rope?

Ex. 46. A ladder 25 feet long rests with one end on level ground at a distance of 6 feet from the vert'cal wall. If its other end reaches a window sill, find, from a figure drawn to scale, the height of the window.

Ex. 47. A ladder 40 feet long placed against a wall reaches a point 25 feet above the ground. From a figure drawn to scale find how far the foot of the ladder is from the wall.

Ex. 48. A kite was held by a string 1200 feet long. The string snapped and the kite fell perpendicular to the ground. If it was picked up at a spot 800 feet from the place where I was flying it, find from a diagram how high the kite flew.

CHAPTER XI.

CIRCLES IN AND ABOUT TRIANGLES

I—Circum-Circle.

Ex. 1. Draw a triangle ABC in which $a=2''$, $b=1{\cdot}5''$ and $c=1{\cdot}2''$. Draw the perpendicular bisectors of AB and AC. Let the bisectors cross at S. From S draw a perpendicular SD to BC. How does D divide BC ?

Ex. 2. Draw a triangle ABC in which $a=7$ cm., $\angle B=50^\circ$. $\angle C=70^\circ$. Draw the perpendicular bisectors of AB and AC. Let them meet in S. Join S to D the mid-point of BC. Measure the angles SDB and SDC. What do you notice ?

Ex. 3. Construct a triangle ABC in which $a=5$ cm. $b=3{\cdot}9$ cm. and $c=6$ cm. Draw the perpendicular-bisectors of the sides. They all meet in one point. Call it O. Find how far O is from each vertex. What do you notice ? With O as centre and radius OA describe a circle. Through what all points of the triangle does it pass ?

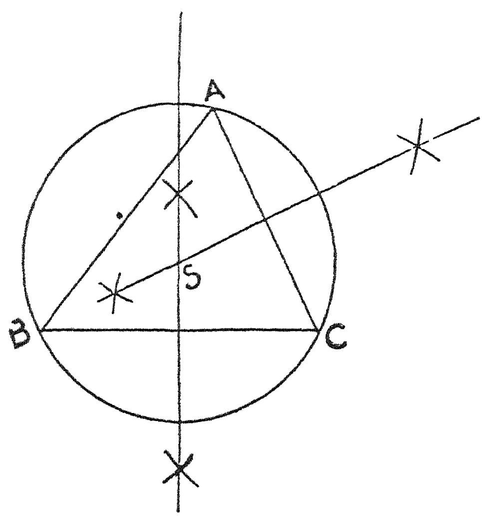
Ex. 4. Construct any six triangles. In each find the point where the perpendicular-bisectors of the sides meet. With that point as centre and the distance of a vertex from it as radius, describe a circle. What do you notice in each case ?

Ex. 5. Suggest a method to describe a circle about a triangle.

The circle passing through the vertices of a triangle is called the *circum-circle* of the triangle. The triangle is said to be *circumscribed.*

CONSTRUCTION XIII.

Describe the circum-circle about a given triangle.



ABC is the given triangle.

Construction—Draw the perpendicular bisectors of any two sides (say AC and BC) of the triangle. Let the bisectors cross at S. With S as centre and radius SA describe a circle. This is the required circum-circle.

Ex. 6. Construct the following triangles and describe the circum-circle about each.

(i) $a=6$ cm., $b=5$ cm., $c=7$ cm.

(ii) $a=2{\cdot}5''$, $\angle B=50°$, $\angle C=80°$.

(iii) $\angle A=80°$, $b=3{\cdot}7$ cm., $c=3{\cdot}5$ cm.

(iv) An isosceles triangle in which the base$=2{\cdot}8''$ and the vertical angle$=70°$.

(v) An isosceles triangle in which the base$=6{\cdot}5$ cm. and perpendicular from the vertex to the base$=7$ cm.

(vi) An equilateral triangle of side 6 cm.

Write your construction in each case.

Ex. 7. Circumscribe a circle about each of the following triangles:

(i) $a=2''$, $\angle B=60°$, $\angle C=56°$.

(ii) $a=2''$, $\angle B=45°$, $\angle C=71°$.

(iii) $a=2''$, $\angle B=37°$, $\angle C=72°$.

Compare the radii of the three circles. What do you notice? What is the magnitude of the vertical angle in each case?

Ex. 8. Describe a circumcircle about the triangle ABC, in which $a=2''$, $\angle B=57°$, $\angle C=50°$. Is the radius of this circle equal to those in exercise 5? What is the magnitude of the vertical angle of this triangle?

Ex. 9. Construct a triangle ABC having $\angle A = 60°$, $b = 5·5$ cm., $c = 6$ cm. Describe a circle about it. Let the perpendicular bisector of BC cut the arc on the side opposite to A at X. Join AX. Measure $\angle$BAX and $\angle$CAX. What do you notice?

Ex. 10. Construct a triangle ABC in which $b = 3·5''$, $\angle A = 70°$, $\angle B = 50°$. Find O the centre of the circum-circle. Join O to A, B and C. Measure and compare the following sets of angles.

(i) $\angle$BOC and $\angle$A.

(ii) $\angle$AOC and $\angle$B.

(iii) $\angle$AOB and $\angle$C.

What do you learn from the above comparisons?

Ex. 11. Describe an equilateral triangle ABC each side being 3 cm long. On BC, CA and AB describe three more equilateral triangles PBC, QCA and RAB respectively. Describe a circle about PQR.

Ex. 12. Describe a circum-circle about the triangle ABC in which $a = 2·5''$, $b = 1·7''$ and $c = 2''$. Mark a point P on the circumference. From P drop perpendiculars PX, PY and PZ on the sides BC, AB and CA respectively (produced if necessary). Show from your construction that X, Y and Z lie in the same straight line.

II—IN-CIRLCE.

Ex. 13. Draw an angle AOB. Draw OX the bisector of the angle. Mark any point P in OX.

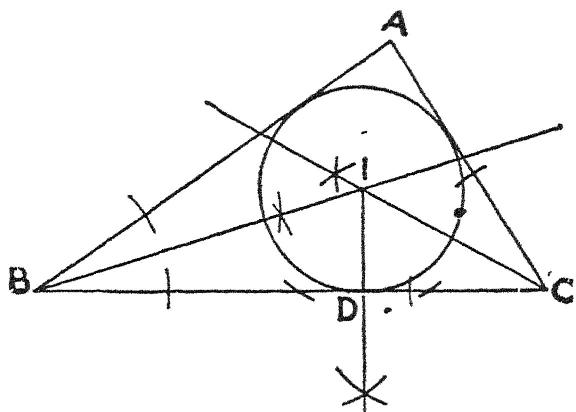
From P draw PC perpendicular to OA and PD perpendicular to OB. Measure PC and PD. What do you notice? With centre P and radius PC describe circle.

Ex. 14. Draw any triangle ABC. Show by drawing that all the bisectors of the angles pass through the same point. Call this point I. From I drop perpendiculars to the sides. Measure the lengths of the perpendiculars. What do you notice? With I as centre and radius equal to one of the perpendiculars describe a circle. What do you observe?

A circle drawn within a triangle touching all the sides is called an *in-circle*. The circle is said to be *inscribed* in the triangle.

CONSTRUCTION XIV

Describe an in-circle in a given triangle.



ABC is the given triangle.

Construction :—Draw the bisectors of any two angles of the triangle. Let the bisectors meet in I. From I draw a perpendicular ID to BC. With I as centre and radius ID describe a circle. This is the required in-circle.

Ex. 15. Construct the following $\triangle$s and inscribe circles within them:—

(i) $a=3{\cdot}9$ cm., $b=6$ cm., $c=5$ cm.

(ii) $a=2{\cdot}3''$, $\angle B=75^\circ$, $c=1{\cdot}8''$.

(iii) an isosceles triangle having the equal sides each 5 cm., long, and the angle between them equal to 30°.

(iv) an equilateral $\triangle$ on a side $=6$ cm.

(v) a right-angled triangle having the hypotenuse $=7$ cm. and one side $=3$ cm.

Ex. 16. Draw two parallel straight lines AB and CD. Draw another straight line to cut AB at P and CD at Q. Bisect one pair of interior angles on the same side of the cutting line. Let the bisectors meet in O. With O as centre and radius equal to the perpendicular from O to one of the lines describe a circle. Describe similarly another circle to touch the three lines.

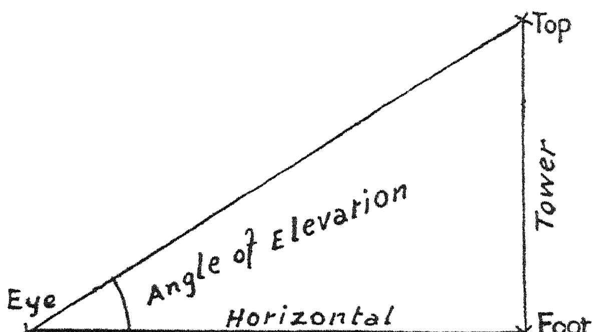
Ex. 17. Construct a $\triangle$ ABC in which $b=7$ cm., $\angle A=60^\circ$, $\angle C=80^\circ$. Produce AB and AC to X and Y respectively. Bisect the angles CBX and BCY. Let the bisectors meet in O. By joining AO show that it passes through the centre of the inscribed circle.

CHAPTER XII.

HEIGHTS AND DISTANCES

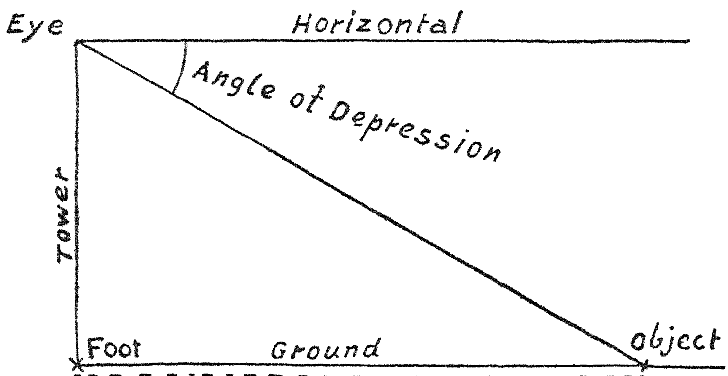
The *vertical line* is the direction taken up by a plumb line *i. e.*, a thread hanging with a weight at the lower end.

If we stand in an open field and look round us, the earth and the sky seem to meet at some distance. This seemingly bounding line is called the *horizon*. The line drawn from the position of the observer's eye to any point on the horizon is a *horizontal line*. The horizontal line is perpendicular to the vertical. If one is looking at a tower, the line drawn from the position of the eye at right angles to the tower is the *horizontal line* (*the line of sight.*)



The angle which the horizontal line make with the line joining the position of the observer's eye to the top of the tower, is called the *angle of elevation* of the top.

If we are at the top of a tower and view an object on the ground, then the angle which the horizontal makes with the line joining the position of the eye to the object, is called *the angle of depression*.



(In the following exercises where a scale for the diagram is not given suggest one yourself and draw the figure.)

Ex. 1. The top of a vertical pole 30 feet high is viewed from a point on the ground 20 feet away from the pole. Find the angle of elevation of the top.

Ex. 2. In the following table the heights of the vertical objects, and the points from where the tops are viewed are given. Find the angle of elevation in each case.

Give a separate scale for each.

Height in feet	80′	65′	35′	60′	55′
Distance of the observer from the object	50′	45′	30′	60′	37′

Ex. 3. A cliff is 1000 feet high. What is the angle of elevation of the summit as viewed from a boat 1000 feet from the cliff?

Ex. 4. The height of a tree is 150 feet. If you observe its top from a place 200 feet from the foot what is the angle of elevation of the top?

Ex. 5 Find the angle of elevation of the sun when a man 6′ high casts a shadow 8 feet long.

Ex. 6. A pole 15 feet high casts a shadow 5′ long. What is the angle of elevation of the sun?

Ex. 7. The angle of elevation of the top of a vertical pole from a point 50 feet from its foot is 50°. What is the height of the pole?

Ex. 8. I observe the top of a clock-tower from a point 40 yards from its foot, and find the angle of elevation to be 60°. Find the height of the tower.

Ex. 9. From a point 100 feet from the foot of a tree I see a bird seated at the top. If the angle of elevation is 45°, what is the height of the tree?

Ex. 10. The top of a flag-staff gives the angle of elevation to be 40° when observed at a distance of 180 feet from its base Find its height.

Ex. 11. I find the angle of elevation of a bird flying to be 55°. All of a sudden it dies and falls vertical to the ground. I walk straight 100 yards and find the bird. At what height was it flying?

Ex. 12. When the angle of elevation of the sun is 50°, the shadow cast by a vertical tower is 40 feet long. Find the height of the tower.

Ex. 13. A lamp post casts a shadow 40 feet long. If the elevation of the sun is 60°, what is the height of the post?

Ex. 14. A cocoanut tree casts a shadow 10 feet long when the elevation of the sun is 70°. Find the height of the tree.

Ex. 15. A kite is held with a string 50 yards long. If the angle of elevation of the kite is 45°, at what height is it flying?

Ex. 16. Standing at the foot of a sloping hill I find the angle of elevation of its top to be 30°. I walk up to its top, a distance of 2,500 yards. At what height do I stand in the end?

Ex. 17. A kite with a string 200 yards long gets stuck up at the top of a tree. I find the angle of elevation of the kite to be 20°. What is the height of the tree? At what distance from the tree do I stand?

Ex. 18. From the foot of a hill I observe the angle of elevation of a fort at its top to be 30°. The length of the road from the foot of the hill to the fort is 1,500 yards. Find the height of the hill.

Ex. 19. From a point 200 feet from the foot of a tower I find the angle of elevation of its top to be 18°. The angle of elevation of the top of a flagstaff on it is 25°. Find the height of the tower and the length of the flagstaff.

Ex. 20. From a distance of 1000 feet the angle of elevation of a cliff is 30°. From the same place the angle of elevation of a tower on the top of the cliff is 35°. Find the height of the tower and the height of the cliff.

Ex. 21. The angle of elevation of the top of an unfinished tower from a point distant 150 feet from its base is 45°. How much higher must the tower be raised so that its angle of elevation from the same point may be 60° ?

Ex. 22. The angle of elevation of the top of a tower is 30°. On walking 100 yards towards the tower the elevation is found to be 60°. Find the height of the tower.

Ex. 23. The angles of elevation of a tower from two places due West of it are 45° and 30°. How high is the tower if the distance between the places of observation be 100 feet ?

Ex. 24. A pigeon sitting on the roof of a school is observed at an elevation of 30° by a man at the gate, and at an elevation of 50° by one 50 feet in front of the first. How high is the bird seated ?

Ex. 25. From the school gate you observe the top of a building and note the angle of elevation to be 30°. You go 80 feet forward and note the angle of elevation to be half a right angle. Find the height of the tree. (No protractor should be used).

Ex. 26. From my house I observe a bird sitting on the top of a tree to be at an elevation of 10°. I take my gun and walk 700 yards towards the tree. Now as I take my aim I find the bird at an elevation of 31°. What is the height of the tree ?

Ex. 27. Standing at a place north of Charminar I find the angle of elevation of one of the minarets to be 69°. I get back 40 feet and observe the elevation to be 58°. Find the height of the minaret. (Scale 20 feet = ½ an inch).

Ex. 28. From a certain place a man finds the angle of elevation of the top of a church spire to be 60°. He walks back 100′ and then observes the angle of elevation of the top to be 45°.

(i) What is the height of the spire ?

(ii) At what distance from the church did he take his first reading ?

Ex. 29. From a certain village a surveyor observes the angle of elevation of the top of a temple Gopuram in another village to be 50°. He gets back half a mile and observes the elevation to be 30°. What is the distance between the two villages ?

Ex. 30. From my place I observe the summit of a hill to be 20°. I walk back a mile and observe the elevation now to be 10°. How far is my place from the hill ? What is the height of the hill ?

Ex. 31. A man wishing to find the breadth of a river, observes from one bank the angle of elevation of the top of a tower on the opposite bank to be 50°. He then recedes 35 feet and finds the elevation to be 34°. What is the breadth of the river ?

Ex. 32. A scout stands at the edge of a river bund and observes the summit of a cliff on the opposite side (near the water) to be at an elevation of 25°. Then he walks back 100 yards and finds the elevation to be 20°. What is the breadth of the river and the height of the cliff ?

Ex. 33. A boy observes an aeroplane to be at an elevation of 30°. He walks one mile towards the place over which the plane is flying and finds the elevation of the plane to be 60°. How much further must he walk to come directly below the plane ? Imagine the plane stationary.

Ex. 34. A man 5 ft. 6 in. tall standing at a distance 30 feet from a lamp post observes the angle of elevation of a crow at its top to be 40°. How high is the post? (Scale 5 feet = 1 cm.)

Ex. 35. Your friend is 6 feet tall. He stands 12 feet from a lamp post and observes the length of his shadow also to be 12 feet. Find the height of the lamp post.

Ex. 36. A boy 5 feet tall standing at a distance of 100 feet from a Musjid observes a dove on the roof of the building. He finds the angle of elevation of the dove to be 30°. How high is the roof?

Ex. 37. A man 6 feet tall stands at a distance of 60 ft. from a tower and observes the angle of elevation of the top to be 50°. What is the height of the tower?

Ex. 38. From the roof of a house 20 feet high, a man observes the angle of elevation of the peak of a temple Gopuram to be 30°. If the temple is 200 feet from the house, find the height of the *Gopuram*.

Ex. 39. A school boy 5′ tall observes the angle of elevation of a bird by sitting at the highest point of a tree to be 30°. Advancing 50 feet towards the tree he observes the angle of elevation to be 50°. Find the height of the tree.

Ex. 40. A man 6′ tall observes the angle of elevation of the top of a tower from a certain place to be 70°. Getting back 60 feet further away from the tower he observes the angle of elevation to be 30°. Find the height of the tower.

Ex. 41. From the roof of a building 20ft. high a boy observes the angle of depression of a ball on the ground to be 30°. How far is the ball from the building ?

Ex. 42. Standing on the Hussain Sagar bund I observe the angle of depression of a duck swimming in the water to be 20°. How far is the duck from the bund if the depth of the water level be 30 feet ?

Ex. 43. From a bridge 40 feet high I observe the angle of depression of an engine at the station to be 40°. How far is the engine from the bridge ?

Ex. 44, From a tree 500 feet high I see a tiger in a cave which is at a distance of 400 feet from the foot of the tree. At which angle to the horizo ntal should I aim my gun at the animal ?

Ex. 45. Standing on a railway bridge, 120 feet above the rails, the station master observes at a depression of 20° a train steaming towards the bridge. If the train runs at 12 miles an hour, in what time will she enter the bridge ?

Ex. 46. A cricket ball has rolled into a ditch. You notice from the wicket that the angle of depression of the ball is 20°. You advance 20 feet

towards the ball and notice that the angle of depression is 50°. Find the depth of the ditch. You are 6 ft. tall.

Ex. 47. From a certain point on the plain the angle of depression of a ball lying in a ditch is observed to be 10°. On getting 50 feet nearer the angle of depression is found to be 40° more. Find the depth of the ditch. You are 5 ft. tall.

Ex. 48. A motor car is driven at a uniform speed towards a cliff. The angle of depression of the car from the summit of the ceiff when it is 1340 yards from the base of the cliff is 30°. Three minutes later the angle is found to be 60°. Find the speed of the car per hour.

Ex. 49. While flying at a height of 300 feet a man in an aeroplane observes the angle of depression of your school to be 18°. What distance should the plane fly straight to be just over the school ?

CHAPTER XIII

I. QUADRILATERALS

A figure bounded by four straight lines is called a *quadrilateral*.

The bounding lines are its *sides*.

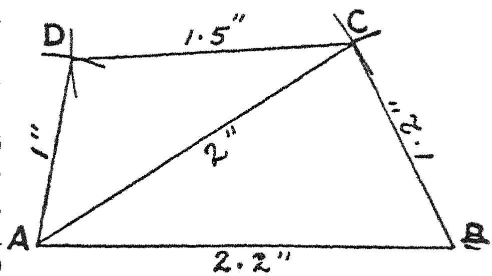
The line joining a pair of opposite vertices is a *diagonal*.

[NOTE :—In all constructions first draw a rough figure. Mark the data. Then the rough figure will suggest the method.]

CONSTRUCTION XV

Construct a quadrilateral ABCD in which AB = 2·2 ", BC = 1·2 ", CD = 1·5 ", DA = 1 ", AC = 2",

Construction :—First construct the triangle ABC in which $a = 1{\cdot}2''$, $b = 2''$ and $c = 2{\cdot}2''$. Then find the point D distant 1·5" from C and 1" from A. Join AD and DC. A B C D is the required quadrilateral.



NOTE :—Constructions of quadrilaterals must always be considered as that of drawing two triangles.

Ex. 1. Construct the following quadrilaterals :—

(i) AB = 6 cm., BC = 5 cm., CD = 4 cm., DA = 5·4 cm., AC = 3·6 cm.

(ii) AB = 1·4″, BC = 1″, CD = 2″, DA = 1·3″, BD = 1·5″.

(ii) XY = 7 cm., YZ = 5 cm., ZW = 4 cm., WX = 6 cm.. XZ = 4·7 cm. The figure is XYZW.

Ex. 2. Describe the quadrilaterals in the following cases :—

(i) AB = 1·6″, BC = 1·3″, ∟B = 60°, AD = 1·8″, DC = 2″.

(ii) AB = 4 cm., ∟A = 70°, AD = 5 cm., BC = 4·5 cm. and DC = 5·6 cm.

(iii) AB = 7 cm., BC = 5 cm., AD = 6·6 cm., DC = 4·7 cm. ∟D = 90°.

(iv) AB = 2″, AD = 1·5″, BC = 2·3″, DC = 2″, ∟C = 100°.

Ex. 3.* Draw quadrilaterals of the following data and find the length of the other diagonal in each.

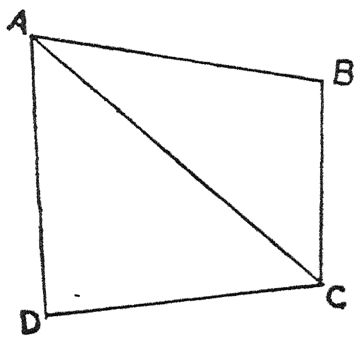
(i) AC = 6·5 cm., AB = 4·3 cm., BC = 5 cm., ∟DAC = 40°, ∟DCA = 45°.

(ii) AC = 2·5″, AD = 2″, DC = 1·3″, ∟BAC = 60°, ∟BCA = 30°.

(iii) BD = 8 cm., BC = 4·7 cm., DC = 5 cm., $\angle$ADB = 58°, $\angle$ABD = 40°.

(iv) AB = 1·6″, AD = 1·5″, DC = 2″ $\angle$CDB = 45°, BCD = 50°.

Ex. 3. In each of the quadrilaterals in exercises 1,2 and 3 find the sum of the four angles. What do you notice?



Ex. 4. ABCD is a quadrilateral. Into how many triangles does a diagonal divide it? What is the sum of the angles in each triangle? What is the total sum of the angles of both the traingles?

What do you learn from this about the sum of the four angles of a quadrilateral?

Ex. 5. Describe the following quadrilaterals. Calculate the fourth angle in each, and verify by measurement.

(i) AB = 5·3 cm., BC = 5 cm., $\angle$B = 110°. $\angle$A = 80° and $\angle$C = 90°.

(ii) BC = 2·2″, DC = 1·5″, $\angle$B = 80°, $\angle$C = 100°. D = 95°.

(iii) AB = 6 cm., AD = 4·5 cm., $\angle$A = 120°, $\angle$B = 85°. $\angle$D = 75°.

(iv) AD = 1·5″, DC = 1·5″, $\angle$D = 130°, $\angle$A = 90°, C = 60°.

Ex. 6. Construct a quadrilateral PQRS in which PQ = 6 cm., PR = 5 cm., $\angle$QPR = 70°, SP = 4·5 cm., SR = 4 cm. Mark A, B, C and D the mid-points of PQ, QR, RS and SP respectively. Join AC and BD. Measure their lengths. What do you notice?

Ex. 7. A quadrilateral field ABCD has the following measurements :—

AB = 400 metres, BC = 300 m., CD = 500m., AD = 350 m., and the diagonal AC = 600m.

Draw a plan of the field (Scale 1 cm. = 100m.) Find the length of the other diagonal.

Ex. 8. ABCD is a quadrilateral field in which AB = 198 yards, BC = 264 yards, AC = 330 yards, DC is perpendicular to AC and is equal to 165 yards. Draw a plan of the field to a scale 1 cm. to 33 yards. How far is D from A?

Ex. 9. The diagonals AC and BD of a quadrilateral plot of ground are 28 yards and 21 yards respectively. The sides BC, CD and DA are 10, 17 and 25 yards respectively. Describe the quadrilateral to a scale 1″ to 10 yards, and find the length of the side AB.

Ex. 10. In a circle of radius 4 cm., draw any quadrilateral ABCD such that the vertices are

on the circumference. Find the sum of (i) $\angle A$ and $\angle C$ (ii) $\angle B$ and $\angle D$. What do you notice?

Ex. 11. Describe half a dozen circles. In each draw a quadrilateral so that the vertices lie on the circumference. In each quadrilateral find the sum of a pair of opposite angles. What do you learn from the results?

Ex. 12. Construct a quadrilateral ABCD in which AB = 5 cm., $\angle B = 80°$, $\angle A = 98°$, AD = 6 cm., $\angle D = 100°$. Find the fourth angle. What is the sum of each pair of opposite angles? Describe a circle about the triangle ABC. Does the circle pass through D also?

Ex. 13. Draw half a dozen quadrilaterals, the sum of a pair of opposite angles in each being two right angles or 180°. Describe circles to pass through any three vertices. Show by drawing the figures that in each case the circle passes through all the vertices of the quadrilateral.

Ex. 14. Describe half a dozen quadrilaterals such that in none the sum of a pair of opposite angles is 180°. Describe circles about any three vertices in each. Do these circles pass through all the vertices? What do you learn from the results of exercises 10, 11, 12 and 13?

Ex. 15. Construct a quadrilateral ABCD in which AB = 7cm., BC = 6 cm., $\angle B = 70°$, CD = 4 cm. and is parallel to AB. Join P and Q the mid-points of AD and BC. Measure PQ and find how much of AB + CD is PQ.

II—TRAPEZIUM

A trapezium is a quadrilateral that has one pair of opposite sides parallel. The other two sides are called the *slant sides*. In exercise 15 ABCD is a trapezium.

Ex. 16. Construct a trapezium A B C D having given :—

(i) BC = 2", CD = 1·5", ∟ C = 85°, DA = 1·2", DA is parallel to BC.

(ii) AB = 3.5 cm., BC = 6 cm., ∟ A = 120°, AD = 4 cm., AD is parallel to BC.

(iii) CD = 2·2", AD = 1·7", ∟ D = 110°, ∟C = 50°. AB is parallel to DC.

(iv) AD is parallel to BC, AB = 5 cm., BC = 6 cm., AC = 4 cm., AD = 4·5 cm.

(v) DC is parallel to AB, AD = 3·5 cm., DC = 4 cm., AC = 5·4 cm., AB = 3 cm.

(vi) AC = 6 cm., BC = 6 cm., AB = 4 cm., AD is parallel to BC, and ∟ACD = 50°

In each of the trapeziums join the mid-points of the slant sides, and find by measurement how much that line is of the sum of parallel sides.

III—PARALLELOGRAMS

A quadrilateral which has its opposite sides parallel is a *parallelogram*.

Ex. 17. Construct a parallelogram ABCD in which AB = 2″, BC = 1·5″, AC = 1·3″. (Draw the triangle ABC. Through A draw a straight line parallel to BC. Through C draw a straight line parallel to AB. Let the parallels meet in D. ABCD is the Parallelogram.)

Ex. 18. Construct the following parallelograms :—

(i) AB = 5 cm., BC = 4·5 cm., $\angle$B = 60°.
(ii) AB = 2″, AD = 1·5″, $\angle$A = 110°.
(iii) BC = 6 cm., CD = 5·5 cm., $\angle$C = 58°.
(iv) AD = 3·7 cm., DC = 6·5 cm., $\angle$D = 70°.
(v) AB = 2·5″, BC = 1·8″, AC = 2″.
(vi) DC = 5 cm., AD = 4 cm., AC = 4·8 cm.
(vii) AD = 2″, AB = 2·3″, BD = 1·5″.
(viii) DC = 7 cm., BC = 6 cm., BD = 5 cm.
(ix) AB = 3·8 cm., BD = 4·9 cm., $\angle$ABD = 40°.
(x) AB = 2·3″, AC = 1·5″, $\angle$BAC = 45°.

Ex. 19. In each of the above parallelograms measure and find the following :—

(i) Lengths of opposite sides.
(ii) The magnitude of opposite angles.
(iii) Draw the diagonals, and find in what ratio the point where they meet divides the diagonals.

PROPERTIES OF A PARALLELOGRAM.

(i) The opposite sides are equal.

(ii) The opposite angles are equal.

(iii) The diagonals bisect one another.

Ex. 20. Using property (i) construct a parallelogram ABCD in which AB = 5 cm., BC = 6 cm., $\angle B = 70°$. (Draw the triangle ABC. Find a point D distant 5 cm. from C and 6 cm. from A. Join AD, AC. ABCD is the required parallelogram).

Ex. 21. Construct the following parallelograms by the method of Ex. 20.

(i) AB = 4 cm., BC = 6 cm., AC = 5 cm.

(ii) AC = 6 cm., $\angle ACB = 60°$, $\angle CAB = 20°$.

(iii) BC = 2", $\angle B = 85°$, $\angle ACB = 30°$.

(iv) AD = 4·8 cm., $\angle DAC = 50°$, AC = 5·5 cm.

Ex. 22. If one angle of a parallelogram be 75°, find the other angles.

Ex. 23. In the following cases one angle of the parallelogram is given, find the other angles (i) 100° (ii) 60° (iii) 70° (iv) 125° (v) 45° (vi) a right angle (vii) one third of a right angle (viii) 130°.

Ex. 24. Construct a parallelogram PQRS in which each of the sides = 5 cm., and one diagonal PR = 6 cm. Draw the diagonals and find at what angle they cut one another. The figure PQRS is called a *rhombus*.

IV.—RHOMBUS.

A rhombus is a parallelogram in which all the sides are equal.

Ex. 25. Construct the following rhombuses:-

(i) side $=5$ cm., one diagonal $=7$ cm.

(ii) side $=2{\cdot}5''$, one diagonal $=2{\cdot}8''$.

(iii) side $=6$ cm., one angle $=80°$.

(iv) side $=1{\cdot}7''$, one angle $=120°$.

(v) side $=4{\cdot}8$ cm., one angle $=$ a right angle.

(vi) side $=2''$, one diagonal $=2''$.

Ex. 26. In each rhombus in Ex. 25 measure the angles which the diagonals make with one another. What do you notice?

Ex. 27. Using the property that the diagonals of a rhombus bisect each other at right angles construct the following rhombuses in which the diagonals are—

(i) 7 cm. and 6 cm.

(ii) $2''$ and $3''$.

(iii) 5.5 cm. and 8 cm.

(iv) $2{\cdot}5''$ and $1{\cdot}7''$

(v) 2 cm. and 9 cm.

V—RECTANGLES.

Ex. 28. Construct a parallelogram ABCD in which AB = 3.7 cm., AD = 5 cm., $\angle$A = a right angle.

Ex. 29. If one angle of a parallelogram is a right angle what is the magnitude of each of the other angles?

A rectangle is a parallelogram in which all the angles are right angles.

Ex. 30. Construct a rectangle ABCD in which AB = 5 cm., and BC = 4 cm. (Draw a straight line AB = 5 cm. At B draw a perpendicular BE = 4 cm. Find a point D distant 5 cm. from C and 4 cm. from A. Join AD, DC. ABCD is the required rectangle). Measure the diagonals.

Ex. 31. Describe the following rectangles in which two sides of each rectangle are given:—

(i) 5 cm., 3 cm. (ii) 1·5″, 1″ (iii) 2″, 1·3″ (iv) 6·8 cm., 4 cm. (v) 7 cm., 4·5 cm. (vi) 6 cm., 5 cm., (vii) 3″, 2·5″ (viii) 2″, 2″ (ix) 8 cm., 8 cm. (x) 1·8″, 1·8″.

Ex. 32. In each rectangle drawn in exercise 3·1, measure and compare the lengths of the diagonals. What do you notice?

Ex. 33. What is the sum of a pair of opposite angles in any rectangle. With the point of meeting of the diagonals as centre and its distance from any vertex as radius describe circles about each of the rectangles in exercise 32. What do you notice?

Ex. 34. Construct a rectangle ABCD in which each diagonal $= 2''$ and one side $AB = 1{\cdot}5''$. (First draw the right-angled triangle ABC in which the hypotenuse $AC = 2''$ and $AB = 1{\cdot}5''$. Then complete the rectangle ABCD).

Ex. 35. Construct the following rectangles in which one side and one diagonal are given:—

One side=	3 cm.	1·7"	4cm.	2·5"	6 cm.	1·5"
Diagonal=	5 cm.	3"	8.5cm	3·3"	8 cm.	2·3"

Describe circles about each rectangle.

VI—Squares.

A rectangle which has all its sides equal is called a *square*.

Ex. 36. Construct squares on the following sides:—(i) 1" (ii) 2·5" (iii) 3" (iv) 3·5" (v) 5" (vi) 1 cm. (vii) 2·4 cm. (viii) 3·5 cm. (ix) 4 cm. (x) 5 cm.

Measure in each the diagonals and the angles which the diagonals make with one another. What do you notice?

Ex. 37. Construct several squares and join the midpoints of the adjacent sides in order. By comparing the sides and angles of each new figure determine what kind of quadrilateral it is?

Ex. 38. Construct a square ABCD in which each side $=7\cdot6$ cm. Divide AB, BC, CD and DA each in the ratio of $1:2$ at P,Q,R and S respectively. Join P,Q,R and S in order. Measure the sides and angles, and find what figure is PQRS.

Ex. 39. In a circle of radius 4 cm. draw two diameters AC and BD at right angles to one another. Join AB, BC, CD and DA in order. Find what figure is ABCD.

Ex. 40. Inscrible squares (one in each) in the following circles of radii: (i) $3\cdot5$ cm. (ii) 5, cm. (iii) $1\cdot5''$ (iv) 4 cm. (v) $1''$.

Ex. 41. In each of the squares in exercise 40 draw perpendiculars to the diagonals at both their ends. What figure is formed in each case?

Ex. 42. Describe squares within and without the circle of diameter (i) $5\cdot5$ cm. (ii) $2''$ (iii) 7 cm. (iv) $2''$ (v) $6\cdot7$ cm.

Ex. 43. Describe a square ABCD each side being $4\cdot5$ cm. At A and C draw perpendiculars to the diagonal AC. Similarly at B and D draw perpendiculars to BD. Produce the perpendiculars both ways meeting one another at P, Q, R and S. Find what figure is formed.

Ex. 44 Describe a square ABCD on a side of 5 cm. Let the diagonals cross at O. With O as centre and radius 3 cm., draw a circle cutting AB at K and L, BC at M and N. CD at P and Q, DA at R and S. Join ML, NP, QR, and SK, and produce them both ways. What is the resulting figure? Show by that the diagonals of both the figures pass through the same point.

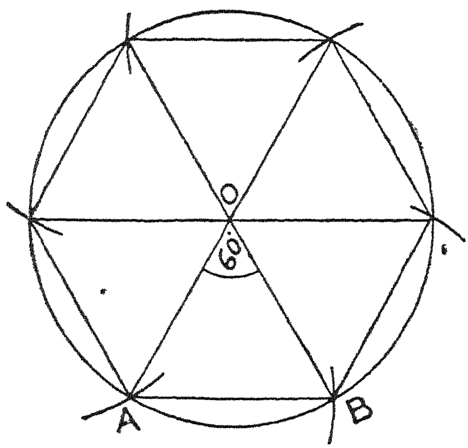
CHAPTER XIV.

REGULAR POLYGONS

A polygon is a figure bounded by more than four sides. What do you call a figure of three sides, and a figure of four sides?

A regular polygon is one which has all its sides and angles equal.

A regular polygon of five sides is called a *pentagon*, that of six sides a *hexagon*, that of eight sides an *octagon* and that of ten sides a *decagon*.



The figure is a regular polygon of six sides inscribed in a circle.

All sides subtend equal angles at the centre (O).

All the angles at O are together equal to four right angles or 360°.

$\therefore$ each side subtends at O an angle $= \frac{360°}{6} = 60°$.

If the figure has n sides, the angle subtended at the centre by each side $= \frac{360°}{n}$.

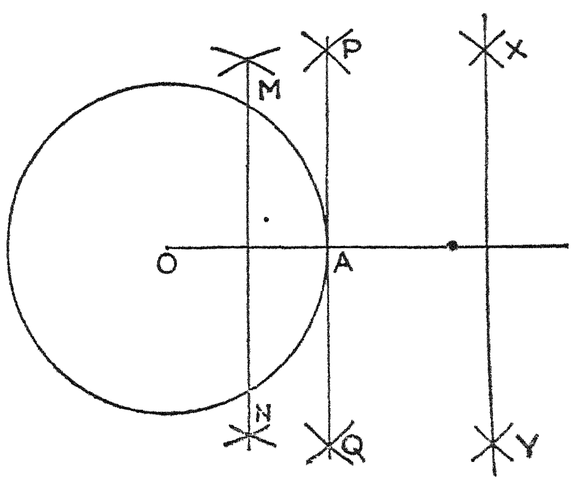
I—General Construction for a Polygon in a Circle

CONSTRUCTION XVI

Inscribe a regular Polygon of x sides in a circle of radius r''.

With O as centre describe a circle of radius r' From O draw two radii OA and OB so that angle AOB $= \frac{360°}{x}$. Join AB. Set off chords round the circumference each equal to AB.

II—A Tangent



OA is the radius of a circle. You will find that the straight line drawn perpendicular to OA at A merely touches the circle. Other perpendiculars either cut the circumference at two points as MN, or do not meet at all as XY.

PA is called the tangent to the circle at A. It does not cut the circle as MN but merely touches it. The tangent is at right angles to the radius at the circumference. So to draw a straight line to touch the circle at a point on the circumference, draw the perpendicular at that point to the radius through it.

III.—POLYGON ABOUT A CIRCLE.

CONSTRUCTION XVII.

Circumscribe a polygon of x sides about a circle of radius r cm.

Construction—In a circle of radius r cm. draw two radii OA and OB (O being the centre) making the angle AOB $= \frac{360}{x}$. Take the length of AB in your compasses, and step off along the circumference. Call the points of division C,D,E etc. Draw tangents to the circle at those points. The resulting figure is the polygon required.

Ex. 1. What is the magnitude of the angle at the centre in a regular figure of (i) six sides (ii) five sides (iii) eight sides (iv) nine sides (v) ten sides (vi) twelve sides (vii) fifteen sides (viii) eighteen sides (ix) thirty sides?

Ex. 2. Describe the following inscribed and circumscribed polygons. (Protractor can be used to draw the angle at the centre).

Number of sides of the polygon.	5	6	8	9	10	12
Radius of the circle	3 cm.	1·7"	5 cm.	5·5cm.	2"	1.5"

Ex. 3. Describe the following polygons in and about the circles with ruler and compasses only.

No. of sides	... ...	6	8	16
Radius	....	4 cm.	2·1"	6 cm.

Ex. 4. In each of the polygons in exercises 2 and 3 draw a perpendicular to the side from its centre. With the length of the perpendicular as radius, and with the centre of the original circle as centre describe a circle. What do you notice in each case?

Ex. 5. In a circle of radius 6 cm. draw two diameters at right angles to one another. Draw bisectors of the angles between them. Join the ends of the four diameters in order. What is the resulting figure?

Ex. 6. In a circle of radius 2" draw two diameters at right angles to one another. Draw bisectors of the angles between them. Draw tangents to the circle at the ends of the four diameters. What is the resulting figure?

Ex. 7. Using exercises 5 and 6 construct octagons in and about the circles of radii (i) 2·3" (ii) 7·5 cm. (iii) 6 cm. (iv) 1·7".

Ex. 8. AB is one side of a hexagon in a circle whose centre is O. OA and OB are joined. What is the magnitude of each of the angles of the $\triangle$OAB? What kind of triangle is OAB? What is the length of a side of the hexagon in a circle of radius x"?

Ex. 9. Use exercise 8 to describe hexagons in circles of diameters (i) 5 cm. (ii) 1·2" (iii) 7 cm. (iv) 1·5".

Ex. 10. Calculate the angle which the radius makes with a side of the polygon and describe (i) a hexagon on a side of 3 cm. (ii) an octagon on a side of 1·6" (iii) a pentagon on a side of 4·5 cm.

CHAPTER XV.

AREAS.

I—Rectangles.

The amount of space enclosed within the bounding lines of a figure is its *area*.

Construct a square on a side = one centimeter. The figure is usually called one centimetre-square. The space enclosed within this square is said to have an area of *one square centimeter*.

Construct a rectangle ABCD in which AB = 8 cm. and AD = 5 cm.

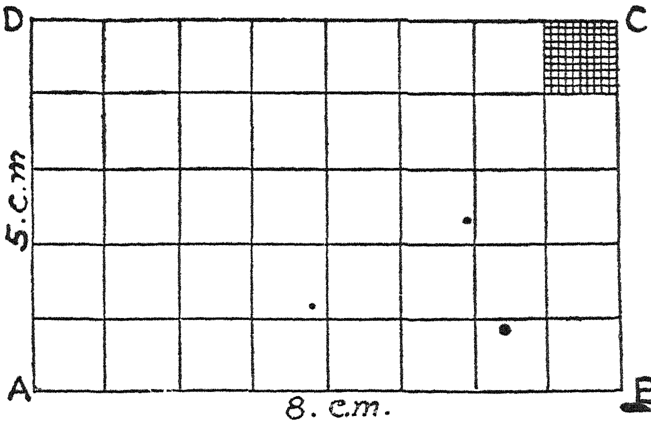


Fig. No. 1

Divide AB into 8 equal parts and through the points of division draw parallels to BC. Similarly divide AD into 5 equal parts and through the points of division draw parallels to AB.

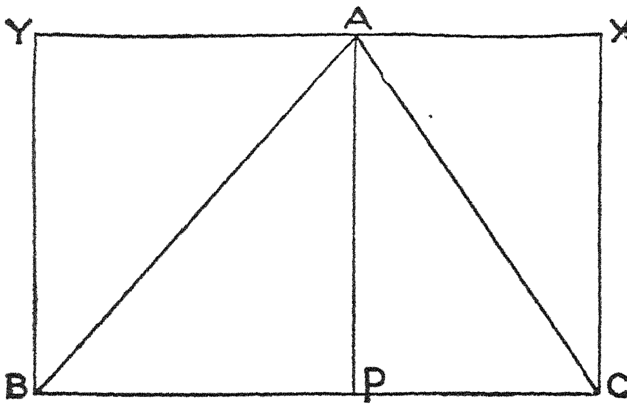


Fig. No. 3

$\triangle$ABC in figure (3) is not right-angled. Construct a rectangle BCXY on BC having the same height AP (the perpendicular from A to BC).

AYBP and AXCP are also rectangles.

Area of the $\triangle$ ABP $= \frac{1}{2}$ rectangle AYBP.

Area of the $\triangle$ ACP $= \frac{1}{2}$ rectangle AXCP.

Adding these, we get,

Area of the $\triangle$ ABC $= \frac{1}{2}$ rectangle BCXY.

$= \frac{1}{2}$ BC $\times$ CX.

$= \frac{1}{2}$ BC $\times$ AP.

$\therefore$ Area of any triangle

$= \frac{1}{2}$ one side $\times$ the perpendicular on it from the opposite vertex.

$= \frac{1}{2}$ base $\times$ altitude (height.)

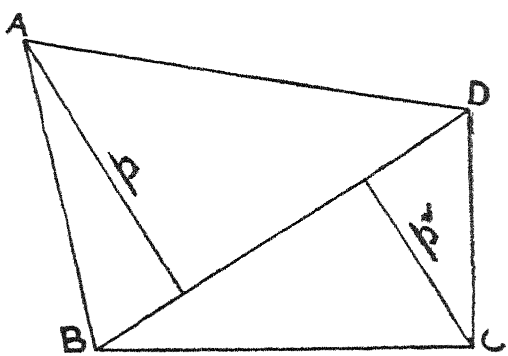
Ex. 3. Find the areas of the triangles in each of which one side and the perpendicular on it from the opposite vertex are as follows:

One side =	10cm.	6″	8 cm.	5″	7·5 cm.
Perpendicular on it =	5 cm.	7″	6 cm.	3″	4 cm.

Ex. 4. Draw the required perpendiculars in exercises 1, 12, 19, 26 of Chapter X and after measuring them calculate the area of each triangle.

III—QUADRILATERALS

A quadrilateral may be considered as two triangles. p and p' are lengths of perpendiculars from A and C on BD.



Area of the quadrilateral ABCD

$=$ Area of $\triangle$ABD + area of $\triangle$ BCD.

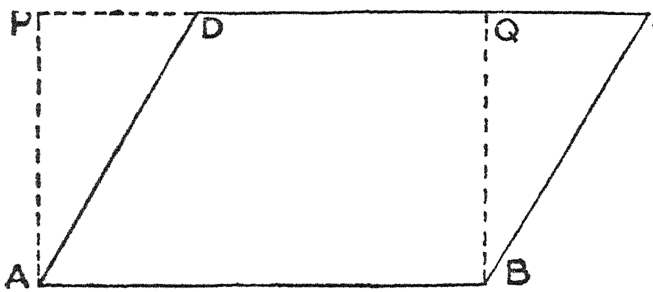
$= \frac{1}{2}$ BD $\times$ p $+ \frac{1}{2}$ BD $\times$ p$'$.

$= \frac{1}{2}$BD (p$+$p$'$).

$= \frac{1}{2}$ the diagonal $\times$ the sum of the perpendiculars on it from opposite vertices.

Ex. 5. Find the areas of quadrilaterals in exercises 1 and 2 of Chapter XIII by drawing perpendiculars from opposite vertices to a diagonal.

IV—Parallelogram



ABCD is parallelogram. Draw AP, BQ perpendicular to DC (or DC produced).

ABQP is a rectangle standing on the same side of AB, and having the same height.

If $\triangle$BQC is cut out and placed on the $\triangle$APD both will be found equal.

Area of parallelogram ABCD

$=$ area of rectangle ABQP.

$=$ AB $\times$ BQ.

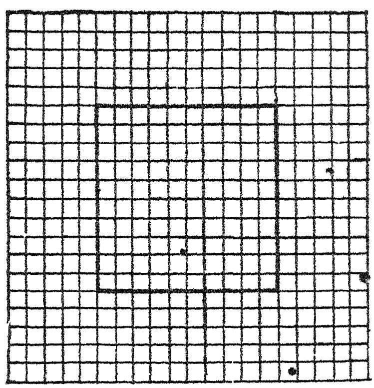
= one side × the perpendicular distance
between it and the opposite parallel.

= base × height.

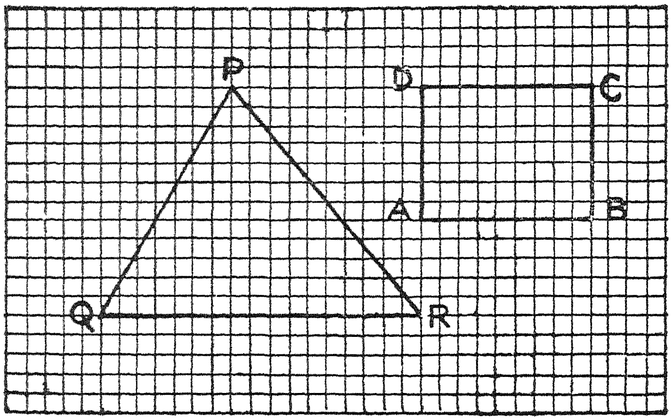
Ex. 6. Find the areas of the parallelograms in exercise 19 and 22 of Chapter XIII.

V—Use of Graph Paper

Ordinary squared graph paper is divided into inch squares, and each inch square is divided into small squares, each side being $\frac{1}{10}$ of an inch. In the figure the square within is an inch square.



One inch-square contains 100 small squares each being a tenth-inch-square. So each small square = 10 of a square inch.



By counting we find that figure ABCD contains 63 small tenth-inch-squares.

$\therefore$ The area of ABCD $= \frac{63}{100}$ of a square inch.
$= \cdot 63$ sq. in.

In figure PQR the outline runs through some of the small squares. In such cases portions of a small square which seem to be one half or more within the figure should be taken as one, and those which are less than one-half should not be counted at all.

Figure PQR contains 102 small squares.

$\therefore$ The area $= \frac{102}{100} = 1 \cdot 02$ sq. in.

VI. Miscellaneous Sums on Areas.

Draw the figures in the following exercises on graph paper and verify your calculation by counting the squares.

Ex. 7. Find the area of a rectangular plot of ground, of which a plan 10 ft. to one inch measures $5''$ by $4''$.

Ex. 8. The badminton court is a rectangle 80 ft. by 40 ft. Draw a plan of the court and find its area.

Ex. 9. Two adjacent sides of a field in the form of a parallelogram are 35 miles and 45 miles. The included angle is $60°$. Draw a plan and find its area. (Scale $1''=10$ miles.)

Ex. 10. The area of a parallelogram ABCD is 6 sq. in. If the base is 4 inches find the height; and construct the parallelogram when the diagonal $BD=2{\cdot}5''$.

Ex. 11. The area of a rhombus is 3·6 sq. in. If each side is $1{\cdot}8°$ find the height and draw the figure. Measure the diagonals and find half their product. What do you notice ?

Ex. 12. The area of a triangle of base $4''$ is equal to that of a square on a side 3 inches. What is the height ? If one base angle is $70°$, construct the triangle.

Ex. 13. What should be the height of (i) a triangle of base $3''$ (ii) a rectangle with one side $3''$ (iii) a parallelogram of base $3''$ so that the area of each may be equal to that of a square on a side $3''$? Draw the figures.

Ex. 14. Draw a quadrilateral ABCD such that AB = 2 , BC = 1·7″, CD = 1″, DA = 1·5″ and one diagonal AC = 2·1″. Make the necessary measurements and calculate its area.

Ex. 15. A quadrilateral field has the following measurements:- BC = 350 yards, AB = 280 yards, CD = 230 yds., AD = 190 yards, and the diagonal AC = 250 yards. Draw a plan and find its area.

VII. CIRCLES.

1. *To measure a curved line*: Make a mark on a piece of cotton thread with ink, and place the thread with the mark at one end of the given line. Then holding it there with the nail of the first finger of the left hand make a short length of the thread coincide with a portion of the line and hold it down with the nail of the first finger of the right hand. Bring the left finger close up to the right finger and proceed as before till the other end of the line is reached. The length of the thread used is then measured by stretching it over a scale.

Draw several curved lines and practise measuring them.

2. *Circumference of a cirle*: Draw a circle of diameter 7 cm. Mark a point P on the circumference. Starting from P lay a thread round the circumference as directed in the above para. Then stretch the thread used over a scale and find the length of the circumference of the circle.

Describe circles of diameters 3·5″, 2·1″, 6·3 cm., 11·2 cm., 10·5 cm., 2·8 cm. Measure the circumferences and tabulate your results thus :—

Diameter.	Circumference.	$\frac{\text{Circumference.}}{\text{Diameter.}}$

You will find that the ratio in the last column is always the same. It is about 3·142 and is usually remembered as the fraction $\frac{22}{7}$. The length of the circumference of a circle $= \frac{22}{7}$ its diameter. To calculate the circumference of a circle multiply its diameter by $\frac{22}{7}$.

Find the length of the circumference of each of the circles of diameters 7″ ; 1·4″; 14 cm. ; 8·4 cm. ; 50 ft. ; 70 yds ; 20 yds ; 30 ft.

3. *Area of a circle* $= \frac{22}{7} \times r^2$. Describe a circle of radius 1·4″ on a graph paper. Find the area of the circle by counting the squares. Then find the value of $\times \frac{22}{7} \times 1{\cdot}4 \times 1{\cdot}4$. The results will approximately be the same.

Find the area of each of the circles of radii: 3·5″, 4·2″, 7 cm., 6·3 cm., 70 ft., 84 yards.

Miscellaneous Exercises.

A

Ex. 1. From a point O draw straight lines OA, OB, OC, OD and OE in different directions. What is the sum of all the angles so formed?

Ex. 2. Draw several pairs of straight lines, so that, in each pair the two lines cut one another. Draw the bisector of one angle in each case and produce it. Find how the bisector in each case divides the vertically opposite angle. What do you learn from these results?

Ex. 3. (*a*) Construct a triangle ABC. Draw a straight line XY=BC. At X make an angle YXZ $= \angle$BAC and XZ =AC. Join YZ. On a thin piece of paper make a tracing of the triangle XYZ and test by applying it to the triangle ABC, such that X falls on A, XY along AB and XZ along AC, whether the tracing can be exactly fitted over the $\triangle$ABC.

(*b*) Construct several pairs of triangles, such that in each pair two sides and the included angle of one triangle are equal to two sides and the included angle of the other, each to each. In each pair test by seeing if a tracing of one triangle can be exactly fitted over the other.

Ex. 4. Construct several pairs of triangles such that in each pair three sides of the one are equal to three sides of the other. As in ex. 3 (*b*) in each pair test by seeing whether a tracing of one triangle can be exactly fitted over the other.

Ex. 5. Construct several pairs of triangles such that in each pair one side and two angles at its ends in one triangle are equal to one side and the correspondings angles in the other triangle. As in the previou exercises test by seeing whether a tracing of one triangle can be exactly fitted over the other.

Ex. 6. How many figures can be drawn if only the three angles of a triangle are given? Can the tracing of any one triangle be exactly fitted over any other? Are the sizes of all triangles so drawn equal?

If a tracing of one triangle can be exactly fitted over another triangle, then the one is said to *coincide* with the other. The two triangles are said to be coincident. *Coincident* or *congruent triangles* are equal in all respects, *i. e.*, three sides and three angles of the one are equal to the corresponding sides and angles of the other triangle.

From the results of the previous exercises state under what all conditions can triangles be equal in all respects.

Ex. 7. Construct a triangle ABC in which $a=2{\cdot}5$ in., $\angle B=67\frac{1}{2}°$ and $c=2$ in. Produce AB and AC to D and E respectively. Draw the bisectors of the angles DBC and ECB. Let them meet in O.

(i) From O draw perpendiculars to the sides of the triangle and measuse their lengths. What do you notice?

(ii) With O as centre and radius equal to any one of the perpendiculars describe a circle. What do you notice?

Ex. 8. Use the method in ex. 7 to draw a circle to touch externally the sides of a triangle ABC in which $a=4$ cm., $b=5$ cm. and angle $C=45°$.

Ex. 9. Draw a triangle ABC in which $b=2$ in., $\angle A=75°$ and $c=2\cdot5$ in. Find D, E and F the mid-points of the sides BC, CA and AB respectively. Find also X, Y and Z, the feet of the perpendiculars from A, B and C to the opposite sides. Show by drawing that the circle passing through D, E and F also passes through X, Y and Z.

Ex. 10. Draw a triangle ABC in which $BC=7$ cm., angle $B=60°$ and $\angle C=45°$. From the vertices draw perpendiculars AP, BQ and CR to the opposite sides. Describe a circle about PQR. Let the circle cut BC, CA and AB at X, Y and Z respectively. Find how X, Y and Z divide the respective sides.

Ex. 11. Describe a circle about a triangle ABC in which $a=1\cdot5$ in., $b=1\cdot7$ in., and $c=2$ in. Mark a point P on the circumference. From P drop perpendiculars PX, PY and PZ on BC, AB and CA respectively (produced if necessary.) Show from your construction that X, Y and Z lie in the same straight line.

Ex. 12. Construct a triangle ABC in which $a=4$ cm., $\angle B=40°$, $\angle C=100°$. Produce BA to any point P. Bisect the angles BAC and CAP. Let

the bisectors meet BC and BC produced in X and Y. On XY describe a semi-circle.

Ex. 13. Construct a quadrilateral ABCD in which AB = 1.5″, BD = 2″, BC = 1.8″ ∟DBC = 30° and ∟ABD = a right angle. Find P, Q, R and S the mid-points of AB, BC, CD and DA respectively. Join them in order and find what kind of figure is PQRS.

Ex. 14. On a side of 1·5″ describe a square. At both ends of each diagonal draw perpendiculars. Produce the perpendiculars till they cut one another. What is the resulting figure? Suggest a method of describing a square about another square.

Ex. 15. On a side of 3″ describe a square. Join the mid-points of the adjacent sides in order. What is the resulting figure? Give a method of describing a square within another given square.

Ex. 16. Describe a circle about an isosceles triangle ABC in which the base BC = 4 cm., and angle A = 36°. Draw the bisectors of the equal angles B and C. Let them cut the circumference again at X and Y. Join AX, XC, AY and BY. What kind of figure is AXCBY?

Ex. 17. Give a method of describing a pentagon on a side of one inch.

Ex. 18 (i) On squared graph paper draw a right angled triangle in which ∟A = a right angle, $b = 3''$ and $c = 4''$ Construct squares on the three sides. Add the areas of the squares on AC and AB, and

compare their sum with the area of the square on the hypotenuse. What do you notice?

(ii) Describe several right angled triangles on graph paper, and compare the area of the square on the hypotenuse with the sum of the squares on the other two sides.

Ex. 19. Find the area of the square on the hypotenuse of each of the following right angled triangles, in which the sides containing the right angle are—

(i) $3''$ and $4''$ (ii) $1''$ and $2''$ (iii) 4 cm. and 5 cm. (iv) 6 cm. and 7 cm. (v) 12 ft. and 5 ft.

Ex. 20. (i) Inscribe a hexagon in a circle of radius $2''$. Join the mid-points of the adjacent sides of the hexagon. What figure is formed? Again join the mid-points of the adjacent sides of the new figure. Find what figure is again formed?

(ii) Give a method of inscribing a hexagon in another given hexagon.

B

Ex. 1. What is the sum of the adjacent angles which one straight line makes with another *on one side of it?*

Ex. 2. What do you known about vertically opposite angles when one straight line cuts another.

Ex. 3. What do you known about angles in the same segment of a circle?

Ex. 4. How many times is the angle at the centre of the angle at the circumference, both standing on the same arc and on the same side of it?

Ex. 5. What is the magnitude of the angle in a semi-circle?

Ex. 6 In what ratio does the perpendicular drawn from the centre of a circle to a chord divide it?

Ex. 7. What angle does a chord in a circle make with the line joining its mid-point to the centre?

Ex. 8. Of all straight lines drawn from a given point to a given straight line which is the shortest?

Ex. 9. How will you find the distance of a point from a given straight line?

Ex. 10. When two straight lines are parallel and a third line cuts them, what do you know about:—

(i) pairs of alternate angles,

(ii) pairs of corresponding angles,

and (iii) the sum of a pair of interior angles on the same side of the cutting line?

Ex. 11. If a straight line cuts two other straight lines, under what all conditions can the two straight lines be parallel?

Ex. 12. A point P is always at the same distance from a given straight line. On what line does P lie ?

Ex. 13. What is the sum of the three angles of any triangle ?

Ex. 14. (*a*) What is the condition that a triangle may be described with three given lenghts as sides ?

(*b*) Can the sum of any two angles of a triangle be less than the third side ?

Ex 15. What do you know about the base angles of an isosceles traingle ?

Ex. 16. How does the bisector of the vertical angle of an isosceles triangle divide the base.

Ex. 17. On what line does the point which is always equidistant from two given points lie ?

Ex. 18. What is the magnitude of each angle of an equilateral triangle ?

Ex. 19. If from the mid-point of the side of a triangle a straight line is drawn parallel to another side, how does it divide the third side ?

Ex. 20. If a straight line joins the mid-points of two sides of a triangle, how is it related to the third side ?

Ex. 21. What is the method to describe a (i) abo[illegible]t (ii) within a given triangle ?

Ex. 22. What is the sum of the four angles of a quadrilateral ?

Ex. 23. What is the sum of a pair of opposite angles of a quadrilateral through the vertices of which a circle passes ?

Ex. 24. What do you know about a pair of opposite angles in a parallelogram.

Ex. 25. How do the diagonals of a parallelogram divide each other ?

Ex. 26. Are the diagonals of a rectangle equal or unequal ?

Ex. 27. At what angle do the diagonals of a rhombus cut each other ?

Ex. 28. How will you describe a regular figure of n sides in a circle of radius r'' (i) within the circle (ii) about the circle ?

Ex. 29. Give the formula for finding the area of (i) a rectangle (ii) a parallelogram (iii) a triangle (iv) a quadrilateral (v) a trapezium (vi) a cicle.

Ex. 30. What do you know about the area of the square on the hypotenuse of a right angled triangle, and the sum of the areas of squares on the other two sides ?

Premier Press, Secunderabad.

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